Lecture No. 17

Standard Galerkin Solutions to the Convection-Diffusion Equation

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial u}{\partial x}\right)$$

- For the standard Galerkin method, the weighting function equals the interpolating functions.
- The standard Galerkin technique is also referred to as the Bubnov-Galerkin method.
- Typically, we use the FEM only to resolve the differential spatial dependence.

  This leads to:

$$\underline{M}\underline{u}_{,t} + \left(\underline{A} + \underline{B}\right)\underline{u} = \underline{P}$$

- Now, we apply the FD method to resolve the remaining differential time dependence. Using a weighted implicit/explicit scheme we have:

$$\left\{\underline{M} + \Delta t\theta\left(\underline{A}_{j+1} + \underline{B}_{j+1}\right)\right\}\underline{u}_{j+1} = \underline{M}\underline{u}_j + (\theta - 1)\Delta t\left(\underline{A}_j + \underline{B}_j\right)\underline{u}_j + \Delta t\theta\underline{P}_{j+1} + \Delta t(1-\theta)\underline{P}_j$$

Linear Finite Elements

Applying linear elements we will obtain the following general nodal equation:

$$\frac{1}{6}\left\{\frac{u_{i+1,j+1}-u_{i+1,j}}{\Delta t}+4\frac{u_{i,j+1}-u_{i,j}}{\Delta t}+\frac{u_{i-1,j+1}-u_{i-1,j}}{\Delta t}\right\}$$

$$\frac{V}{2\Delta x}\{\theta[u_{i+1,j+1}-u_{i-1,j+1}]+(1-\theta)[u_{i+1,j}-u_{i-1,j}]\}$$

$$-\frac{D}{\Delta x^2}\{\theta[u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}]+(1-\theta)[u_{i+1,j}-2u_{i,j}+u_{i-1,j}]\}=0$$

Lumping, we have the following nodal equation:

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}+\frac{V}{2\Delta x}\{\theta[u_{i+1,j+1}-u_{i-1,j+1}]+(1-\theta)[u_{i+1,j}-u_{i-1,j}]\}$$

$$-\frac{D}{\Delta x^2}\{\theta[u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}]+(1-\theta)[u_{i+1,j}-2u_{i,j}+u_{i-1,j}]\}=0$$

This equation for the lumped linear FE formulation is identical to that obtained for the cntral difference FD formulation.

<u>Comparisons between consistent finite element (FE) and lumped finite elements FE-L/FD solutions</u>

<u>A.</u> <u>Fully explicit</u> $\theta = 0.0$

<u>1.</u> <u>Pure Convection</u> $\mathbf{P_e} = \infty$

i. Fig. L29.1a 1/2: Amplitude Portraits

- Shows that there is no damping for either FE-L/FD or FE solutions for any $\mathbf{C}_\#$ values.
- Thus both methods are unconditionally unstable at $\mathbf{P_e} = \infty$.

  Recall that stable solutions require $|\xi_n'| \leq 1$.
- FE seems to be more severely unstable (i.e. it will grow unstable more quickly).

ii. Fig. L29.1b 1/2: Ratio Portraits

- Shows that $|\xi_n'|/|\xi_n'| > 1 \ \forall \ \mathbf{C}_\#$ values. Thus the numerical solution is under-damped as compared to the analytical solution.
- Recall that a perfect solution has a ratio = 1 and an overdamped solution has ratio << 1.

iii. Fig. L29.1b 1/2: Phase Portraits

- Shows that FE has superior phase propagation characteristics as compared to FE-L/FD. Therefore the numerical dispersion properties are better and the wiggles are less severe.
- Recall perfect solution has phase lag = 0.

iv. Fig. L29.2a 1/2: Computed Solution at $\mathbf{C}_{\#} = 0.1, \mathbf{P_e}\ \infty$

- Both solutions show statically unstable behavior.
- Instability grows more quickly in the FE solution than FE-L/FD solution
- These features are predicted by the amplitude portrait.

v. Fig. L29.2b 1/2: Computed Solution at $\mathbf{C}_{\#} = 1.1, \mathbf{P_e}\ \infty$

- Dynamically unstable behavior for both solutions
- Predicted by amplitude portrait

Note: Table L29.1: defines error measures given on all computed results

2. <u>Convection/Diffusion:</u> $\mathbf{P_e} = 2$

i. Fig. L29.3a 1/2: Amplitude Portrait

- Indicates that FE-L/FD is stable for $\mathbf{C_\#} \leq 1.0$ at $\mathbf{P_e} = 2$
- FE solution seems to be unstable for $\mathbf{C_\#} \geq 0.5$ (or lower ?) at $\mathbf{P_e} = 2$. It is stable for $\mathbf{C_\#} = 0.1$
- For stable regions the solutions will be damped

ii. Fig. L29.3b 1/2: Ratio Portrait

- Solutions are for the most part underdamped
- FE-L/FD and FE are very similar except at $\mathbf{C_\#} = 0.1$, the FE-L/FD underdamps whereas FE overdamps but only at high wave numbers.

iii. Fig. L29.3c 1/2: Phase Portrait

- Indicates that FE-L/FD has excellent phase propagation characteristics except at $\mathbf{C_\#} = 0.1$.

- FE also has good phase propagation characteristics except at $\mathbf{C}_{\#} = 0.5$ where there is a phase lead.

iv. Fig. L29.4a 1/2: Computed Solution at $\mathbf{P_e} = 2$ and $\mathbf{C}_{\#} = 0.1$

- Both solution are good, FE is slightly superior over FE-L/FD.
- Slight overall phase lag for FE-L/FD and slight phase lead for FE as indicated by the phase portraits.
- Underdamping is not a problem as is indicated by Ratio portraits
- Any wiggles introduced will be damped out due to amplitude damping as indicated by amplitude portrait (physical damping present).

v. Fig. L29.4b 1/2: Computed Solution at $\mathbf{P_e} = 2$ and $\mathbf{C}_{\#} = 1.1$

- Dynamically unstable
- FE much more so than FE-L/FD. This was indicated by amplitude portraits.

3. General Conclusions FE-L/FD and FE explicit

- Stability is a severe problem for both FE-L/FD and FE schemes when an explicit time discretization is used. Always unstable for convection dominant cases.
- FE has more restrictive ability characteristics than FE-L/FD. Also unstable behavior for FE is more severe (but this really doesn't matter).
- Diffusion dominant low $\mathbf{C}_{\#}$ problems handled reasonably well (less restrictive $\mathbf{C}_{\#}$ conditions for FE-L/FD than FE).

B. Crank-Nicolson $\theta = 0.5$

1. Pure Convection: $\mathbf{P_e} = \infty$

i. Fig. L29.5a 1/2 and L29.5b 1/2: Amplitude Ratio Portraits

- Shows no damping and perfect analytical damping for all $\mathbf{C}_{\#}$ values for both FE-L/FD and FE
- Thus no stability constraints and no under or over damping.
- Thus amplitude and ratio values are exact or perfect.

ii. Fig. L29.5c 1/2: Phase Portraits

- Shows that FE has <u>much</u> better phase propagations characteristics than FE-L/FD. This is especially true of lower $\mathbf{C}_{\#}$ values ($\mathbf{C}_{\#} = 0.1, 0.5$) although it is in general the case.

iii. Fig. L29.6a 1/2: Computed Solution at $\mathbf{P_e} = \infty$ and $\mathbf{C}_{\#} = 0.1$

- FE solution has fewer and much smaller wiggles
- FE solution has a peak which is well propagated while FE-L/FD solution has severe peak phase lag.
- These differences result from the better phase behavior of the FE scheme.

iv. Fig. L29.6b 1/2: Computed Solution at $\mathbf{P_e} = \infty$ and $\mathbf{C}_{\#} = 1.1$

- FE solution is again better.

  However difference is not as large as at $\mathbf{C}_{\#} = 0.1$ since although phase behavior of FE is better it is not drastically better than FE-L/FD at high $\mathbf{C}_{\#}$ values.

2. <u>Convection/Diffusion:</u> $\mathbf{P_e} = 2$

i. Fig. L29.7a 1/2: Amplitude Portrait

- All cases damp for all wavelengths
- Thus stable and will help damn wiggles

ii. Fig. L29.7b 1/2: Ratio Portrait

- For FE-L/FD, all cases damp less than analytical solution. Thus there is no over-damping whatsoever.
- For FE low $\mathbf{C}_{\#}$ values have some overdamping but only at low wavelengths. High $\mathbf{C}_{\#}$ values behave like the FE-L/FD solution.

iii. Fig. L29.7c 1/2:

- FE has much superior phase properties compared to FE-L/FD.
- FE has slight phase lead behavior at higher $\mathbf{C}_{\#}$ values but only at very low wavelength ratios.

iv. Fig. L29.8

- FE is superior to FE-L/FD.
- FE-L/FD exhibits slight overall phase lag due to poorer phase behavior.

- Because of longer wavelengths involved in this problem, no substantial phase lag problems.
- Also any wiggles initially generated due to phase errors will be damped out due to physical damping present.

v. Fig. L29.8b 1/2: Computed Solutions at $\mathbf{P_e} = 2$ and $\mathbf{C_\#} = 1.1$

- FE again superior to FE-L/FD.
- Both solutions have deteriorated as compared to low $\mathbf{C_\#}$ values, but only slightly so.

3. General Conclusions FE-L/FD and Fe with Crank-Nicolson

- Unconditionally stable for both
- Accuracy is much superior than FE than for FE-L/FD mainly due to much better phase behavior.

  In highly physically damped cases this leads to better overall phase of the distribution (with no wiggles present in either solution).

C. **Fully Implicit** $\theta = 1.0$

1. Pure Convection: $\mathbf{P_e} = \infty$

i. Fig. L29.9a 1/2:

- All wavelengths are damped for $\mathbf{P_e} = \infty$ and $\mathbf{C_\#} = 0.1 \rightarrow 1.1$.
- Thus we can expect stable solutions which are damped for both FE-L/FD and FE.

ii. Fig. L29.9b 1/2: Ratio Portrait

- Numerical solutions are overdamped as compared to analytical solutions.
- For $\mathbf{C_\#} = 0.1$ this overdamping is not severe and is slightly greater at small wavelengths in FE than for FE-L/FD.
- For higher $\mathbf{C_\#}$ values $(0.5 \rightarrow 1.1)$, overdamping is severe for the entire wavelength range and is similar for both FE-L/FD and FE.

iii. Fig. L29.9c 1/2: Phase Portrait

- Shows that FE phase behavior is much better than FE-L/FD. This is especially true for lower $\mathbf{C}_{\#}$ values (0.1 → 0.5).

iv. Fig. L29.10a 1/2: Computed Solution at $\mathbf{P_e} = \infty$ and $\mathbf{C}_{\#} = 0.1$

- FE-L/FD solution is very poor exhibiting wiggles which are not sufficiently damped.
- Overall phase is also very poor.
- These wiggles are due to FE-L/FD having poor phase behavior in important wavelength range while there is not sufficient damping to get rid of them.
- FE solution has no wiggles and overall phase is excellent.
- However the solution is overdamped. This is due to the fact that the phase propagation characteristics are very good and we have sufficient damping for the very short wavelengths. However we've lost parts of our solution through this combined dispersion and damping process and this accounts for the overdamped solution.

v. Fig. L29.10b 1/2: Computed Solution at $\mathbf{P_e} = \infty$ and $\mathbf{C_\#} = 1.1$

- Both FE-L/FD and Fe solutions are severely overdamped. This feature is predicted by ratio portraits.

2. Convection/Diffusion $\mathbf{P_e} = 2$

i. Fig. L29.11a 1/2: Amplitude Portrait

- Shows that both FE-L/FD and Fe always damp and thus have stable behavior for all $\mathbf{C_\#}$ values considered.

ii. Fig. L29.11b 1/2: Ratio Portraits

- Shows that for all higher $\mathbf{C_\#}$ values (0.5 – 1.1) there is severe overdamping in the high wavelength range.
- At $\mathbf{C_\#} = 0.1$, behavior is much better and only the FE shows damping at very low wavelength values.

iii. Fig. L29.11c 1/2: Phase Portrait

- FE has superior phase behavior, especially at lower $\mathbf{C_{\#}}$ values.

iv. Fig. L29.12a 1/2: Computed Solution at $\mathbf{P_e} = \infty$ and $\mathbf{C_{\#}} = 0.1$

- Both FE-L/FD and FE solutions are reasonable
- FE solution has slightly better overall phase.

v. Fig. L29.12b 1/2: Computed Solution at $\mathbf{P_e} = \infty$ and $\mathbf{C_{\#}} = 1.1$

- Both FE-L/FD and Fe are severely overdamped. This is predicted by ratio portrait.

<u>3. General Conclusions FE-L/FD and FE fully implicit</u>

- FE is somewhat superior in that it is slightly more accurate and eliminates wiggles better.
- In general, implicit methods for both FE-L/FD and FE often lead to severely overdamped results.

D. General Conclusions – sections A, B, C

- Best time integration scheme is Crank-Nicolson. Explicit methods were generally unstable for convection dominated flows while implicit methods have severe overdamping problems. Recall that C-N is $0(\Delta t)^2$ accurate whereas explicit and implicit methods were only $0(\Delta t)$ accurate.
- FE is much better than FE-L/FD for C-N in terms of overall quality of results. Quality is excellent for low $\mathbf{P_e}$ and good for high $\mathbf{P_e}$ (some wiggles remained).
- If you're applying consistent FE is it always computationally worthwhile to use C-N since you're backsubstituting into matrices anyway (although matrices may have to be reset).
- Always exercise extreme caution with any numerical solution.

Table L29.1

<u>Error E1:</u> Integral measure of the overall error of the numerical solution.

$$E1 = \frac{1}{m(t)} \left[\int_0^L \left(\phi^{\text{num}}(x,t) - \phi^{\text{ex}}(x,t) \right)^2 dx \right]^{1/2}$$

Value for Exact Solution = 0.0

<u>Error E2:</u> Discrete measure of the overall error of the numerical solution.

$$E2 = \frac{1}{m(t)} \left[\sum_i \left(\phi_i^{\text{num}}(t) - \phi^{\text{ex}}(x_i,t) \right)^2 \right]^{1/2}$$

Value for Exact Solution = 0.0

<u>Error E3:</u> Point measure of the artificial damping of the numerical solution (peak depression).

$$E3 = \left| \frac{\phi_{\max}^{\text{ex}}(t) - \phi_{\max}^{\text{num}}(t)}{\phi_{\max}^{\text{ex}}(t)} \right|$$

Value for Exact Solution = 0.0

Error E4: Point measure of the maximum spurious oscillation in the numerical solution.

$$E4 = \left| \frac{\phi_{\text{max,neg}}^{\text{num}}(t)}{\phi_{\text{max}}^{\text{ex}}(t)} \right|$$

Value for Exact Solution = 0.0

Error E5: Point measure of the phase shift introduced in the numerical solution.

$$E5 = \frac{x_{\text{max}}^{\text{ex}}(t) - x_{\text{max}}^{\text{num}}(t)}{x_{\text{max}}^{\text{ex}}(t)}$$

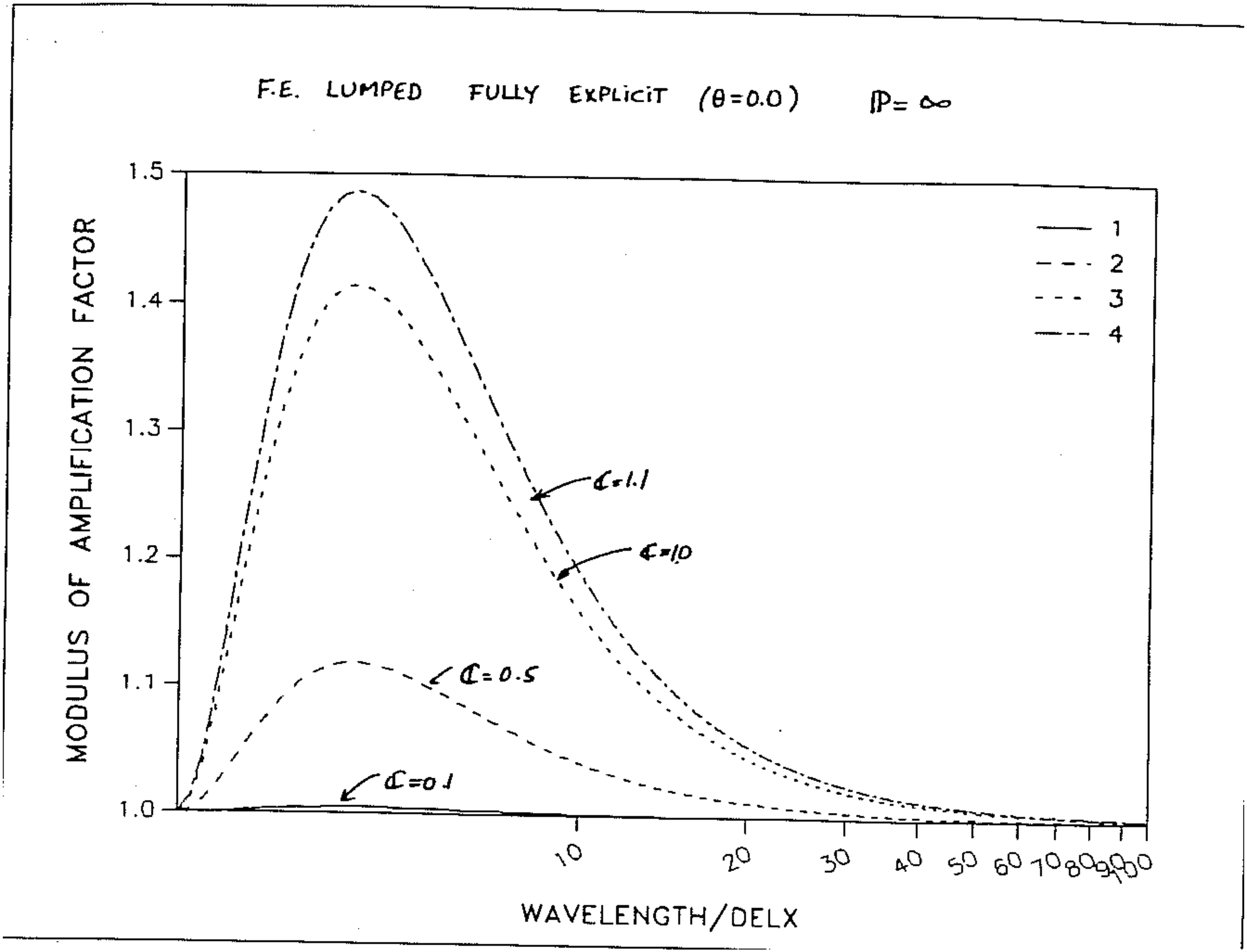
Value for Exact Solution = 0.0

Error E6: Integral measure of mass preservation.

$$E6 = \frac{1}{m(t)} \int_0^L \phi^{\text{num}}(x,t)dx$$

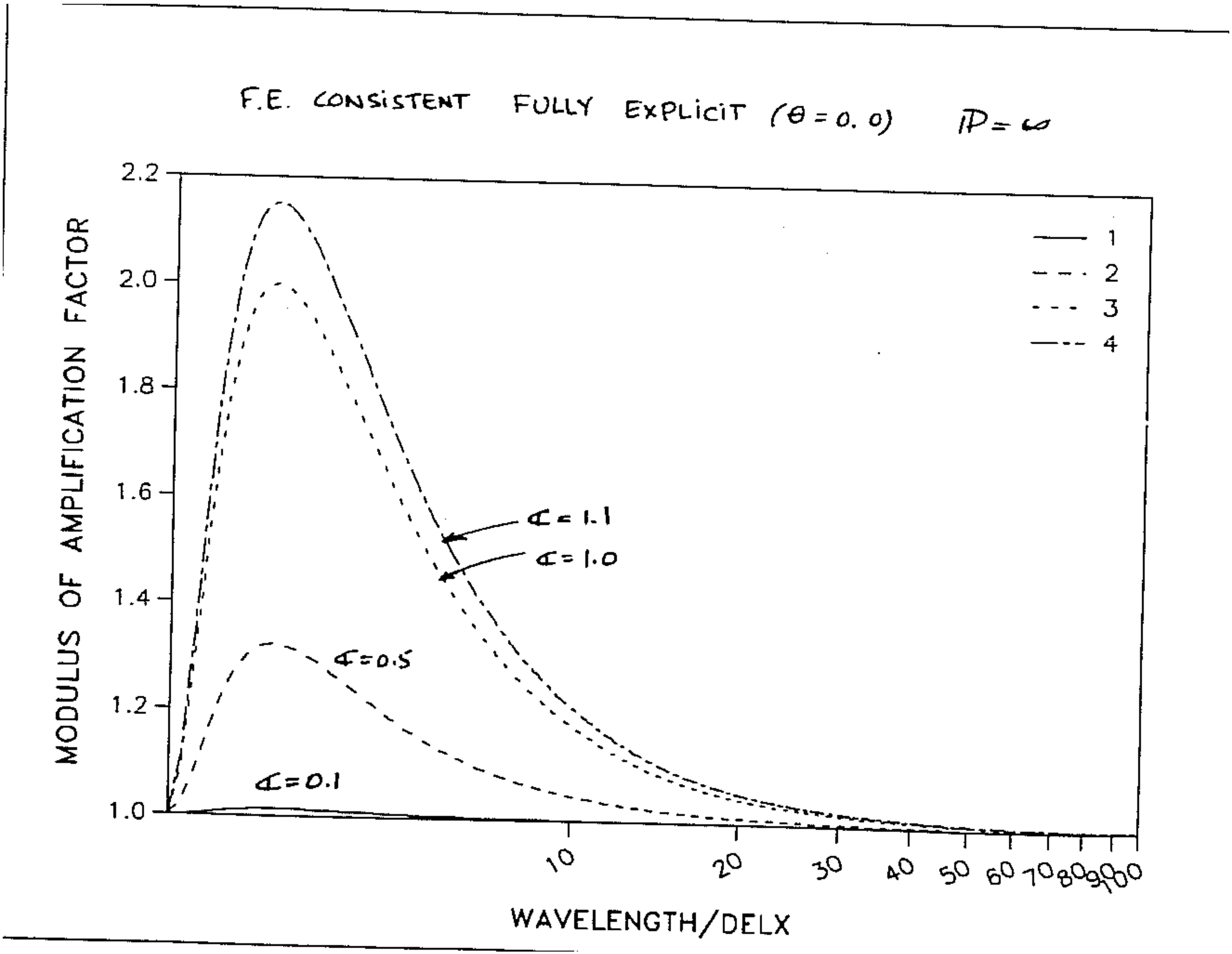
Value for Exact Solution = 1.0

Fig. L29.1a1



F.E. LUMPED FULLY EXPLICIT (θ=0.0) P= ∞
MODULUS OF AMPLIFICATION FACTOR
1.5
1.4
1.3
1.2
1.1
1.0
1
2
3
4
C=1.1
C=10
C=0.5
C=0.1
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.1a2



F.E. CONSISTENT FULLY EXPLICIT (θ = 0. 0) IP = ∞
MODULUS OF AMPLIFICATION FACTOR
WAVELENGTH/DELX
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2.0
1.8
1.6
1.4
1.2
1.0
10
20
30
40
50
60
70
80
90
100
1
2
3
4
C = 1.1
C = 1.0
C = 0.5
C = 0.1

Fig. L29.1b1

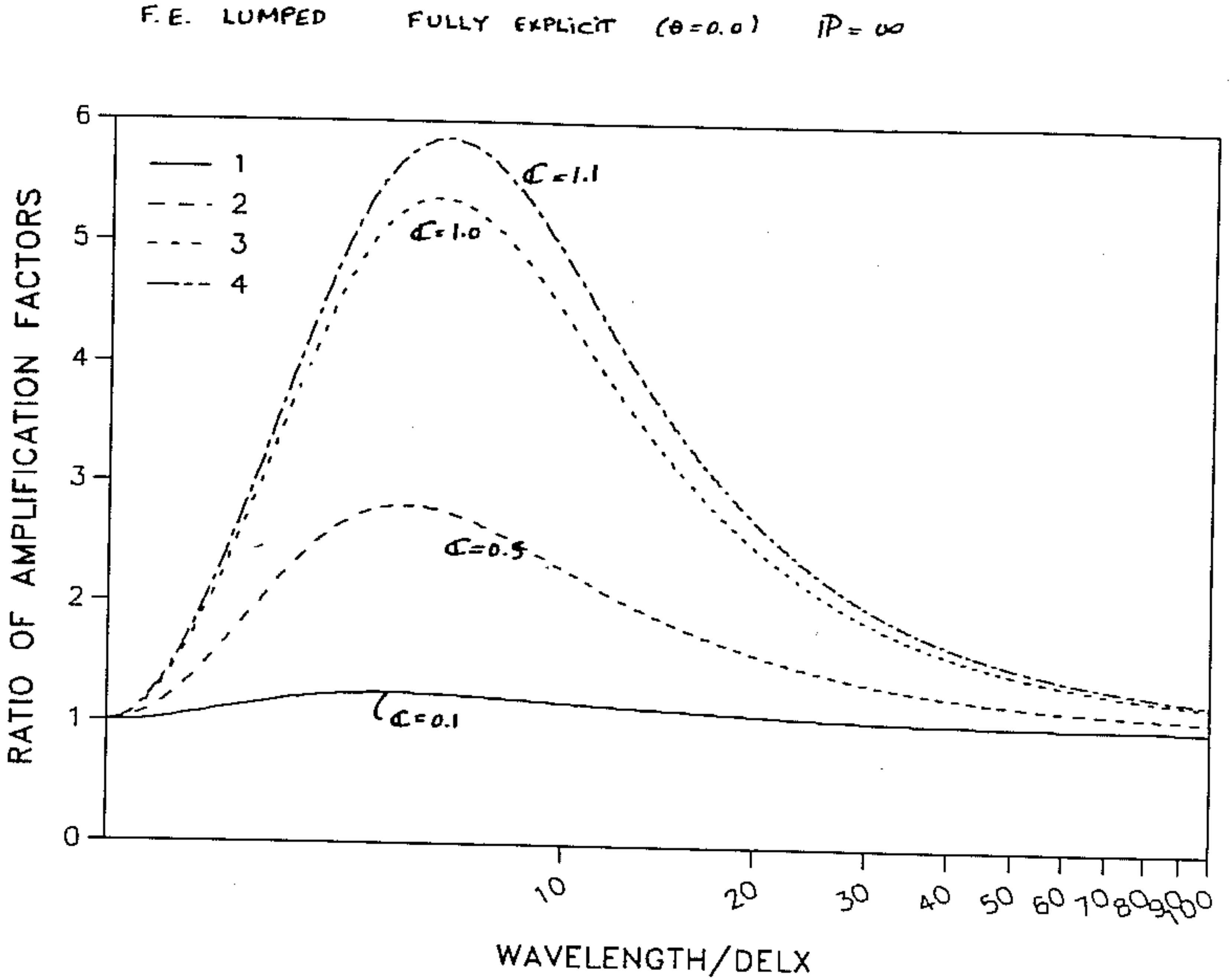
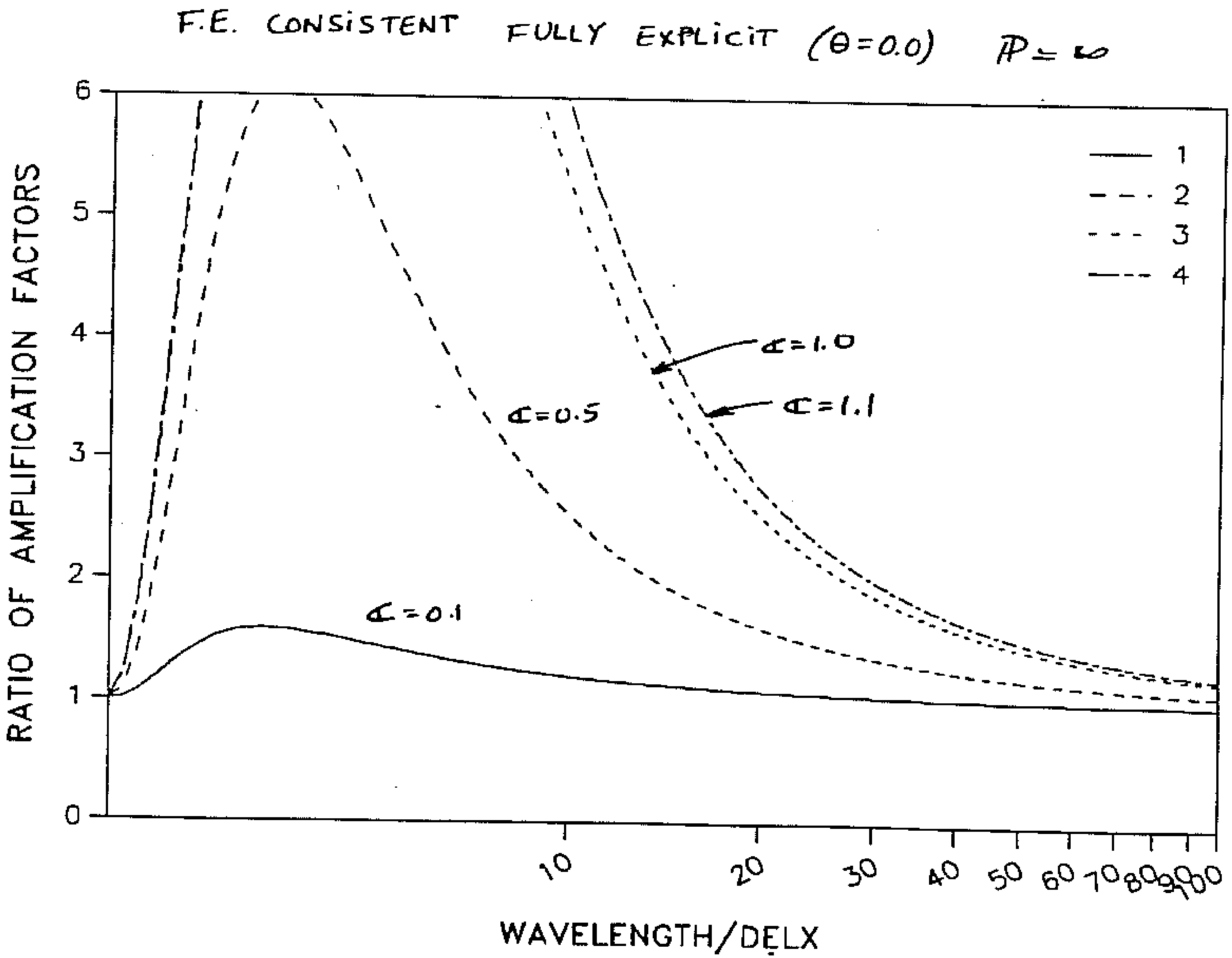
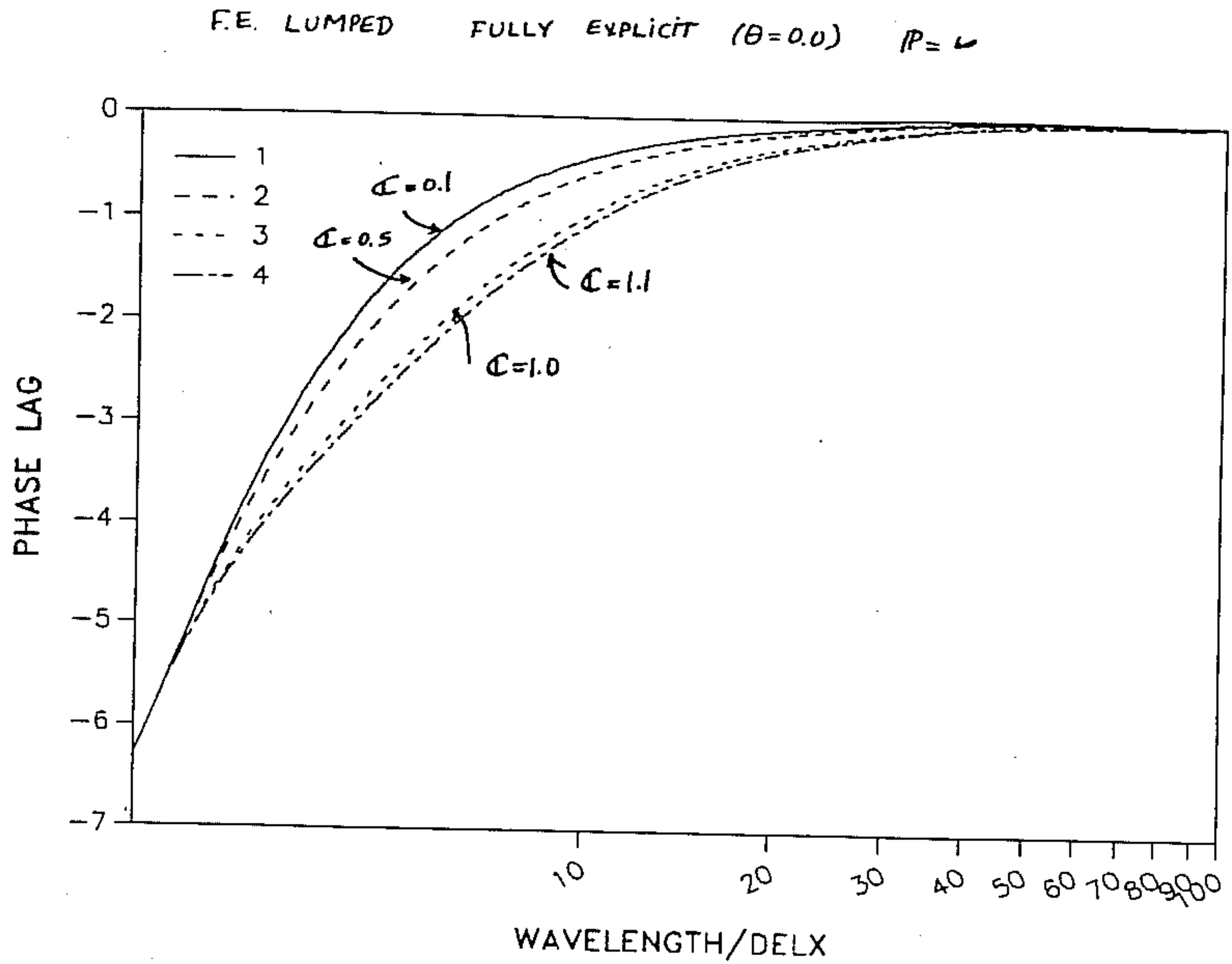

F. E. LUMPED
FULLY EXPLICIT (θ=0.0)
P = ∞
1
2
3
4
C=1.1
C=1.0
C=0.5
C=0.1
RATIO OF AMPLIFICATION FACTORS
WAVELENGTH/DELX
0
1
2
3
4
5
6
10
20
30
40
50
60
70
80
90
100

Fig. L29.1b2



F.E. CONSISTENT FULLY EXPLICIT (θ=0.0) P = ∞
RATIO OF AMPLIFICATION FACTORS
WAVELENGTH/DELX
0
1
2
3
4
5
6
10
20
30
40
50
60
70
80
90
100
1
2
3
4
α=1.0
α=1.1
α=0.5
α=0.1

Fig. L29.1c1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−1
−2
−3
−4
−5
−6
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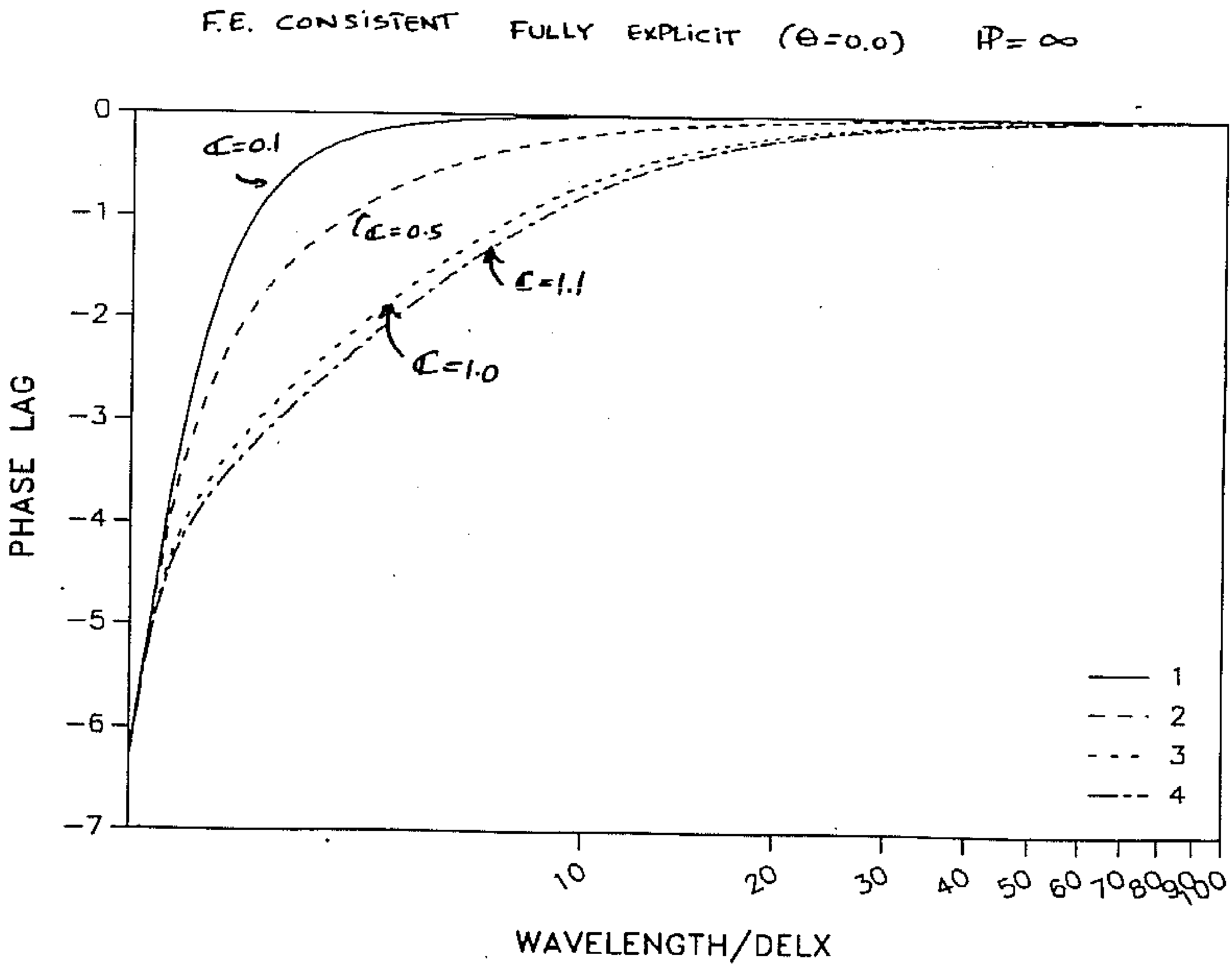
PHASE LAG
1
2
3
4
C = 0.1
C = 0.5
C = 1.1
C = 1.0
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.1c2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PHASE LAG
WAVELENGTH/DELX
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C=1.1
C=1.0
1
2
3
4
0
−1
−2
−3
−4
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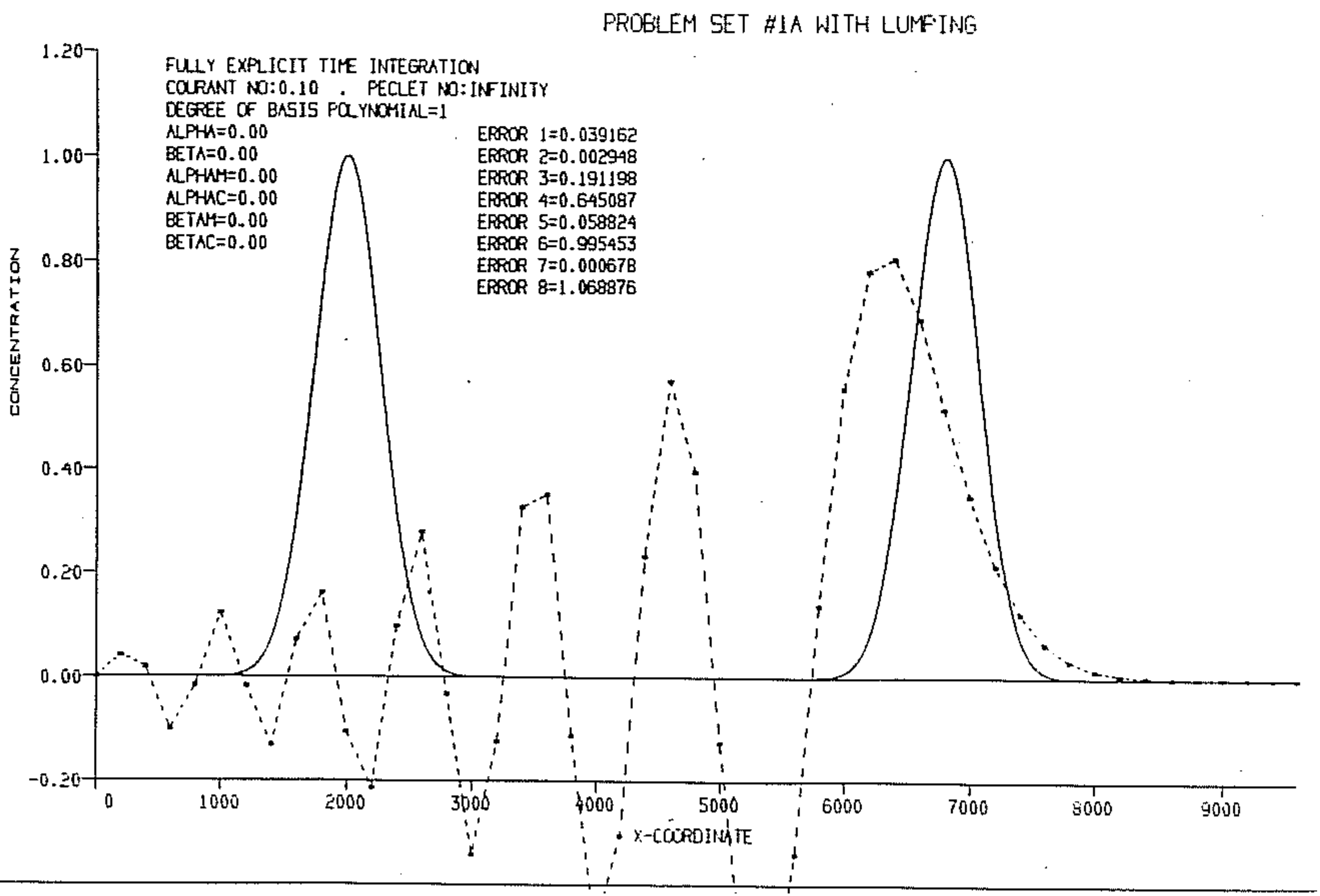
−6
−7
10
20
30
40
50
60
70
80
90
100

Fig. L29.2a1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1.00
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0.60
0.40
0.20
0.00
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2000
3000
4000
5000
6000
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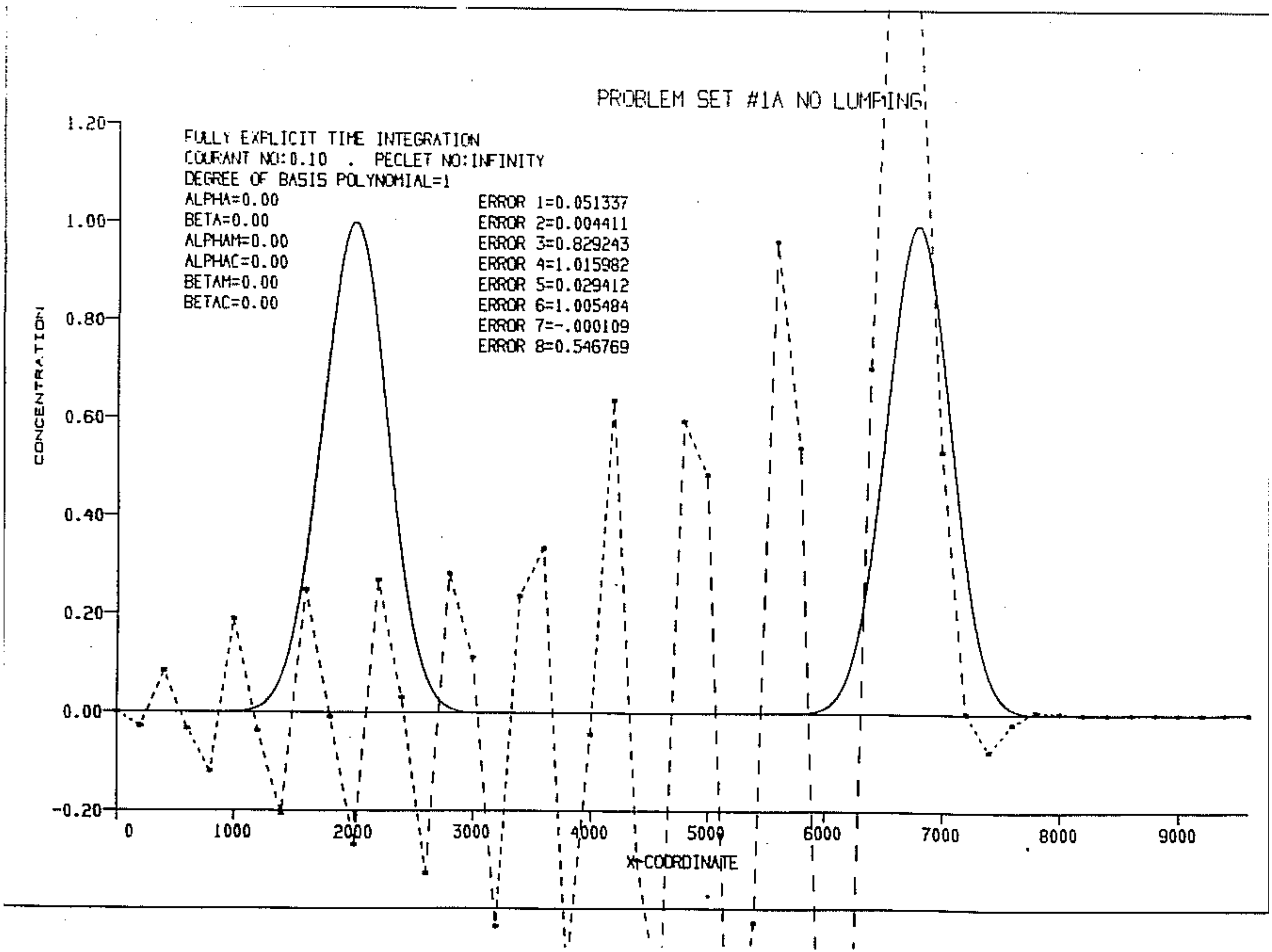
8000
9000
X-COORDINATE

Fig. L29.2a2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1.00
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0.60
0.40
0.20
0.00
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2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.2b1

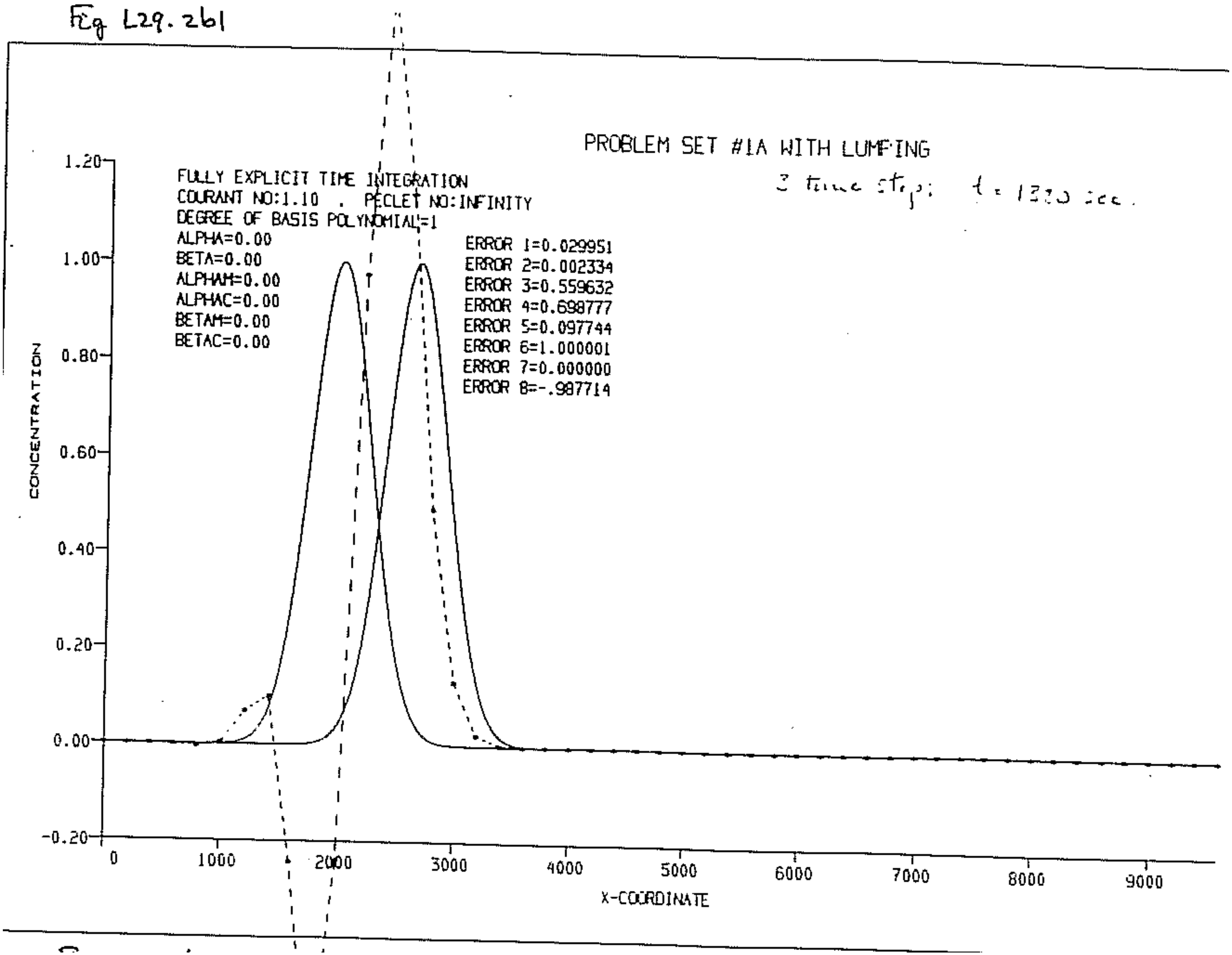

Fig L29.2b1
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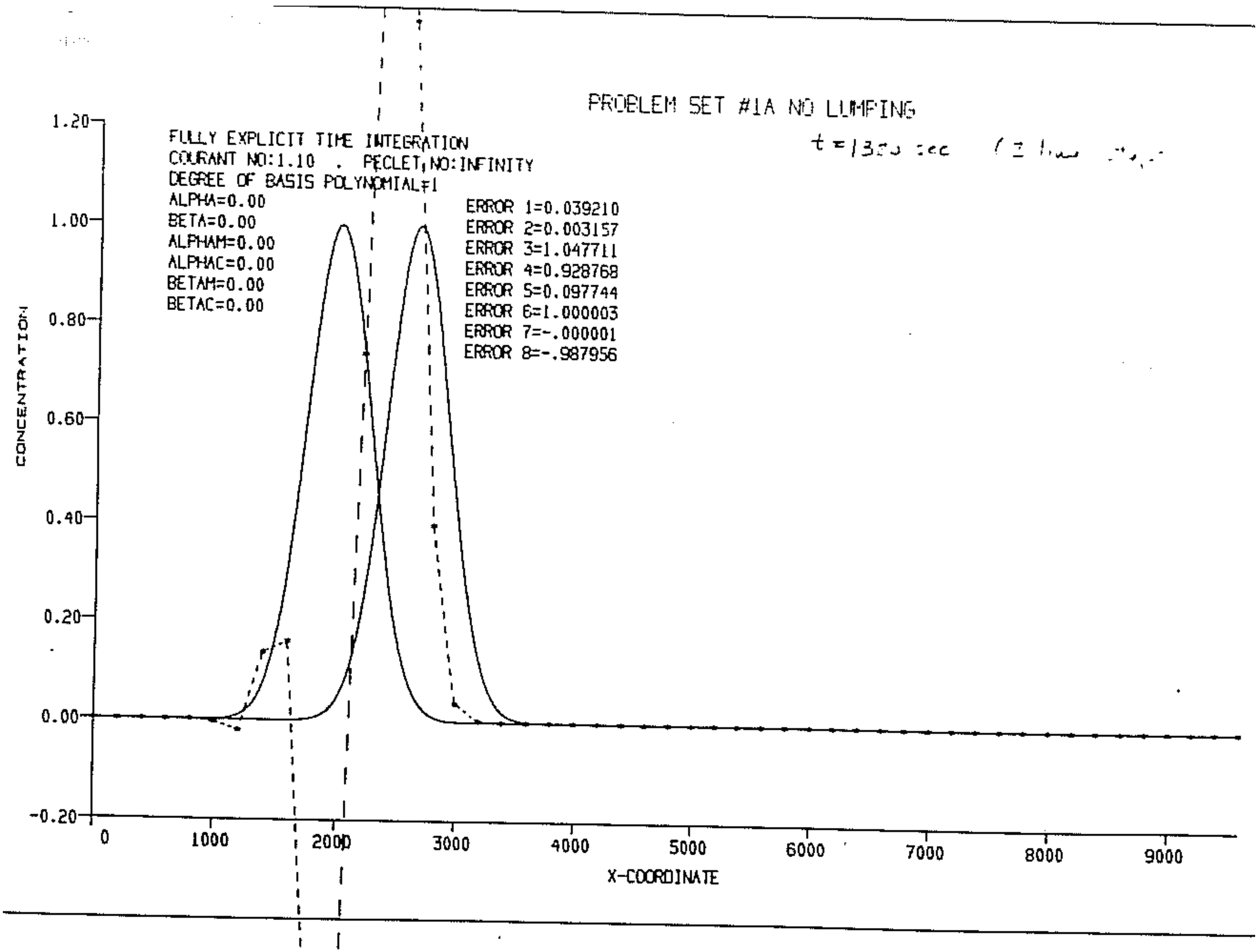
CONCENTRATION
X-COORDINATE

Fig. L29.2b2



PROBLEM SET #1A NO LUMPING
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CONCENTRATION
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1.00
0.80
0.60
0.40
0.20
0.00
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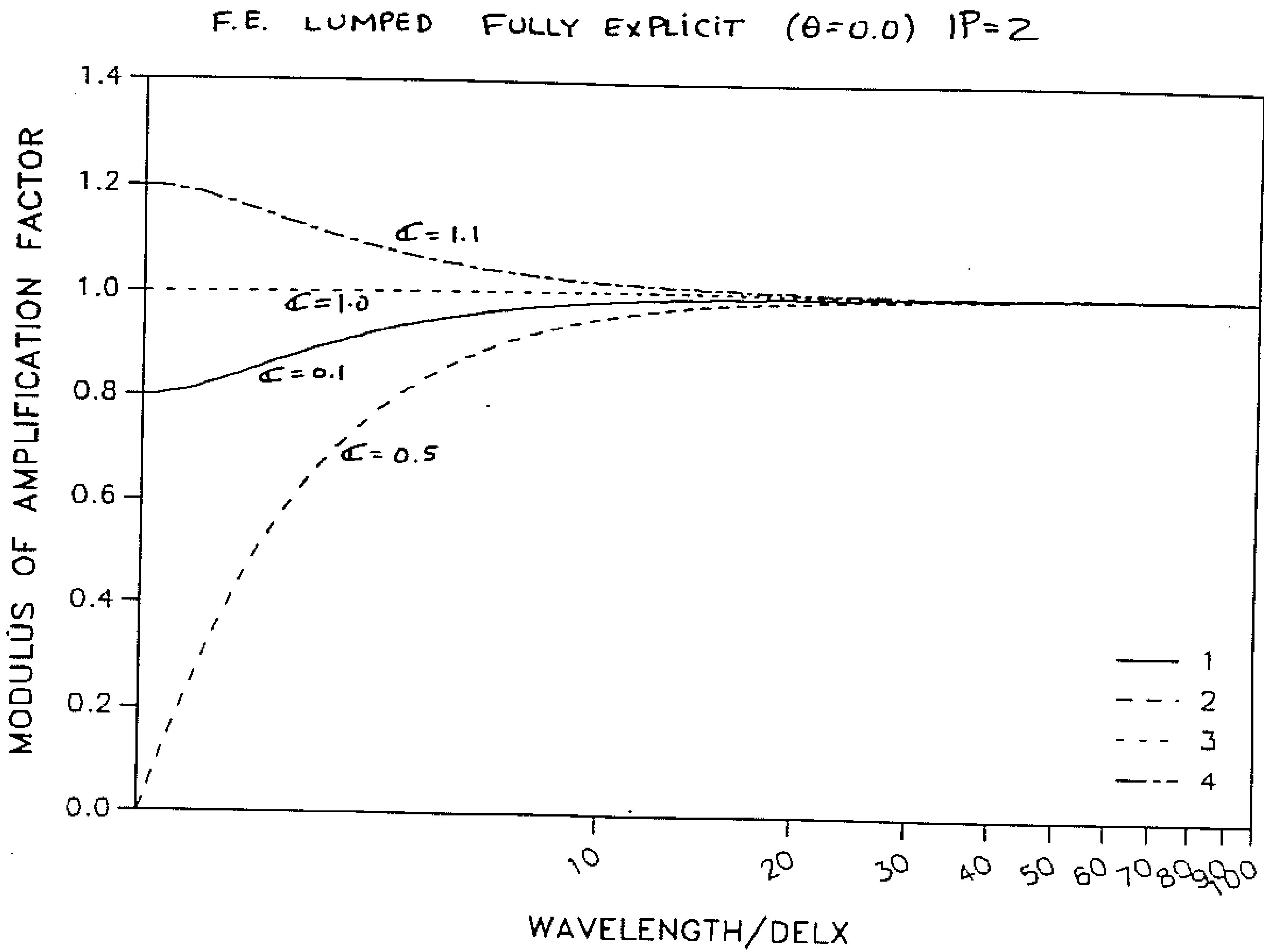
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.3a1


F.E. LUMPED FULLY EXPLICIT (θ=0.0) IP=2
MODULUS OF AMPLIFICATION FACTOR
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
C = 1.1
C = 1.0
C = 0.1
C = 0.5
1
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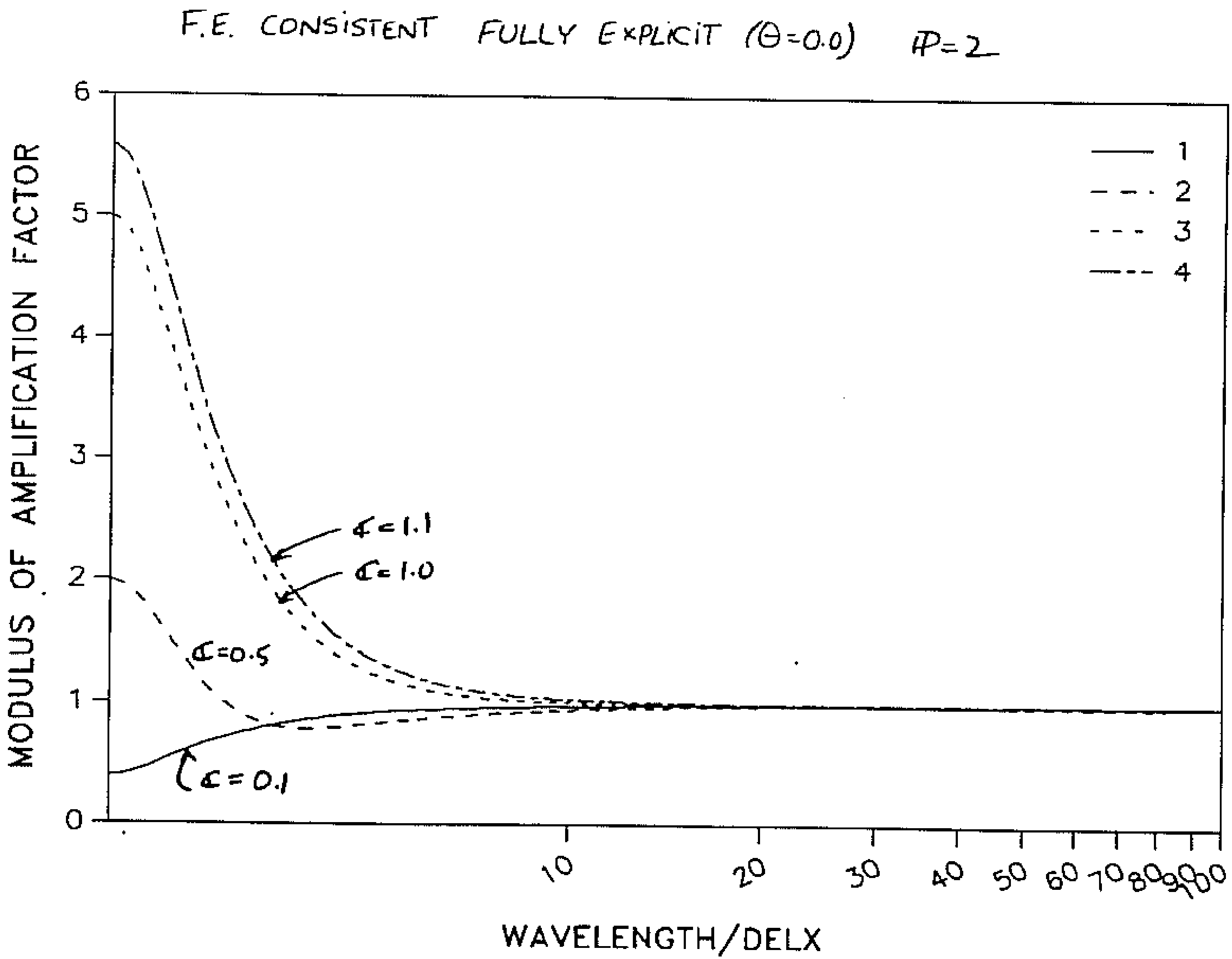
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.3a2



F.E. CONSISTENT FULLY EXPLICIT (Θ=0.0) P=2
MODULUS OF AMPLIFICATION FACTOR
WAVELENGTH/DELX
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3
4
C=1.1
C=1.0
C=0.5
C=0.1
0
1
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4
5
6
10
20
30
40
50
60
70
80
90
100

Fig. L29.3b1

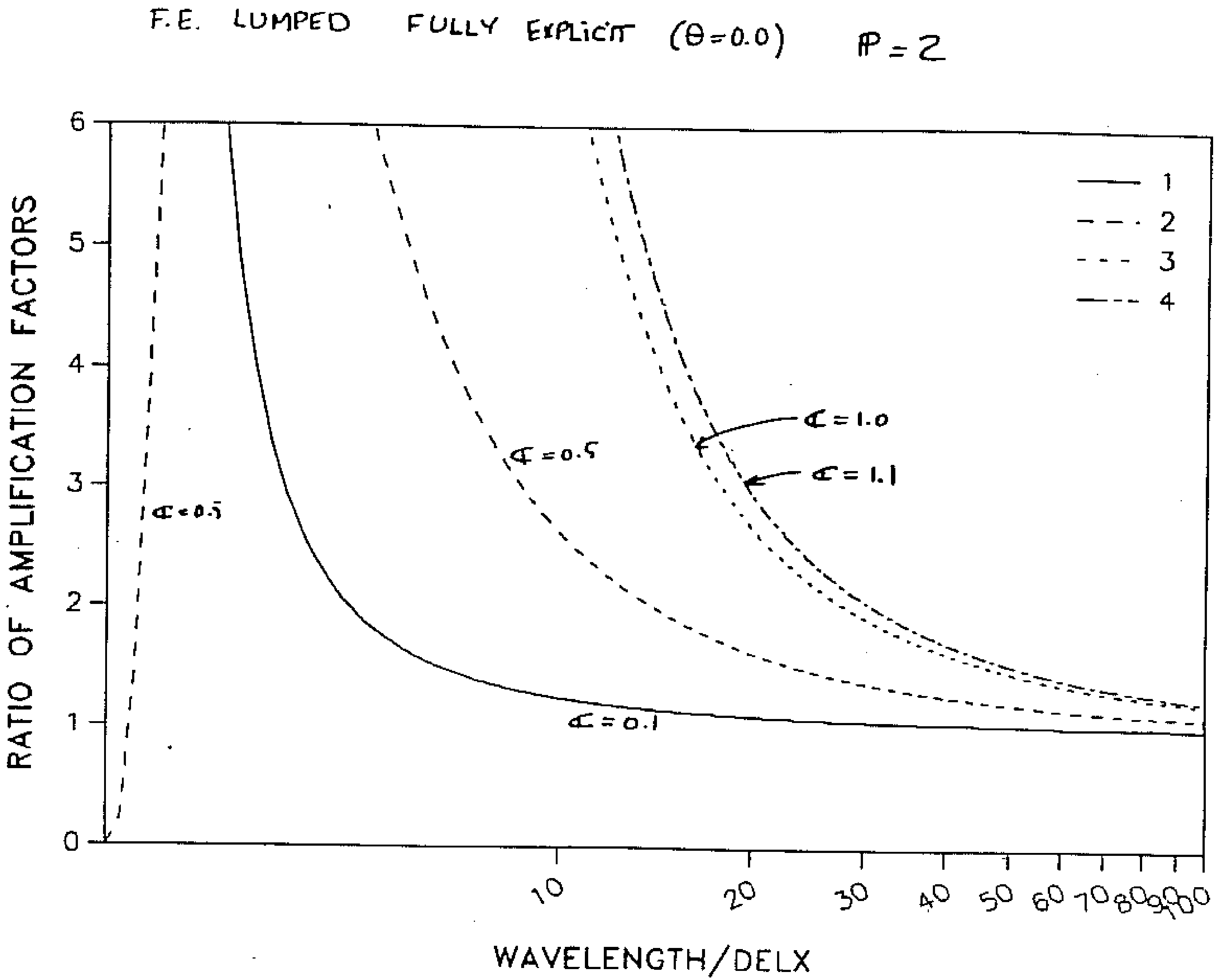

F.E. LUMPED FULLY EXPLICIT (θ=0.0) P=2
RATIO OF AMPLIFICATION FACTORS
WAVELENGTH/DELX
0
1
2
3
4
5
6
10
20
30
40
50
60
70
80
90
100
1
2
3
4
C=1.0
C=1.1
C=0.5
C=0.5
C=0.1

Fig. L29.3b2

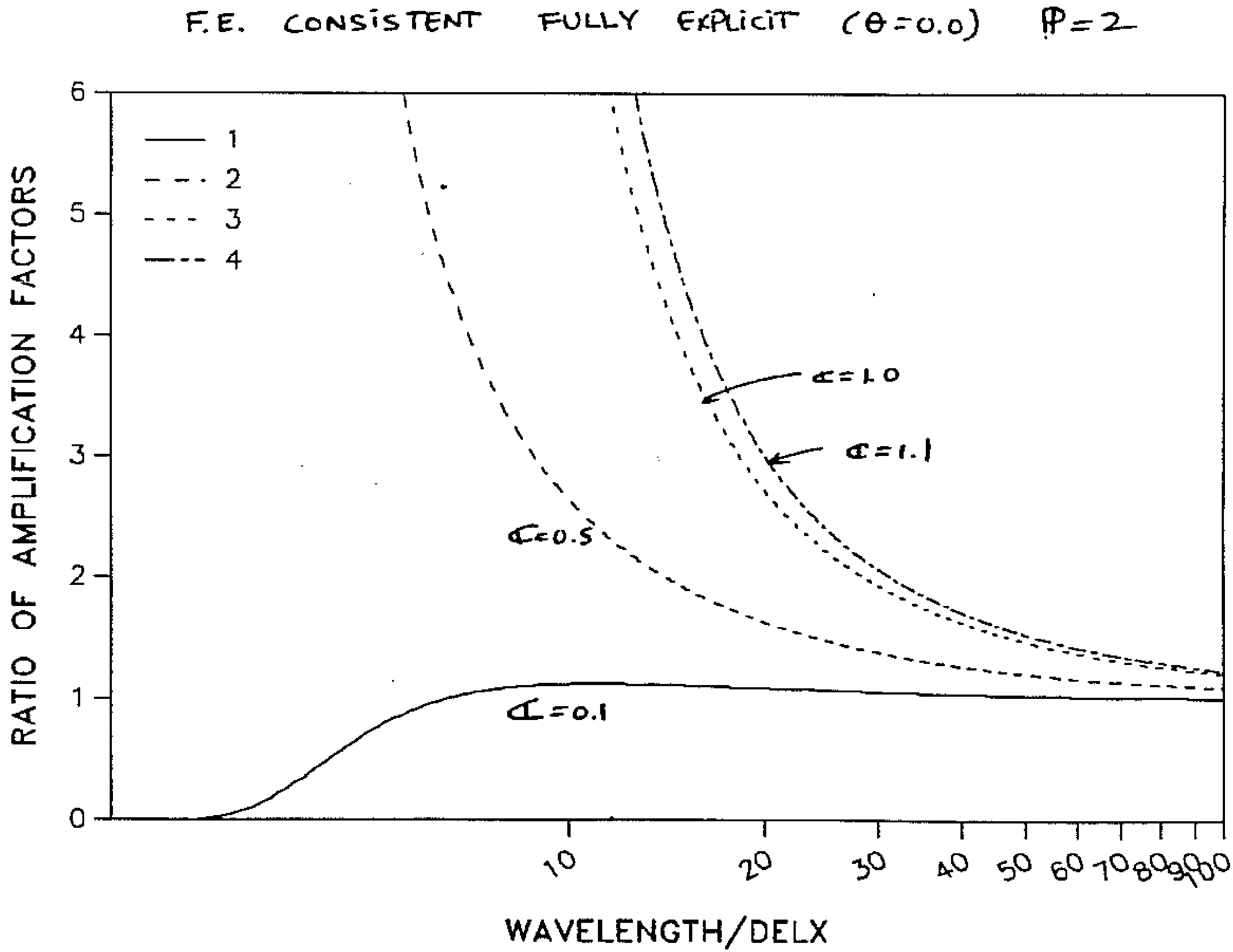
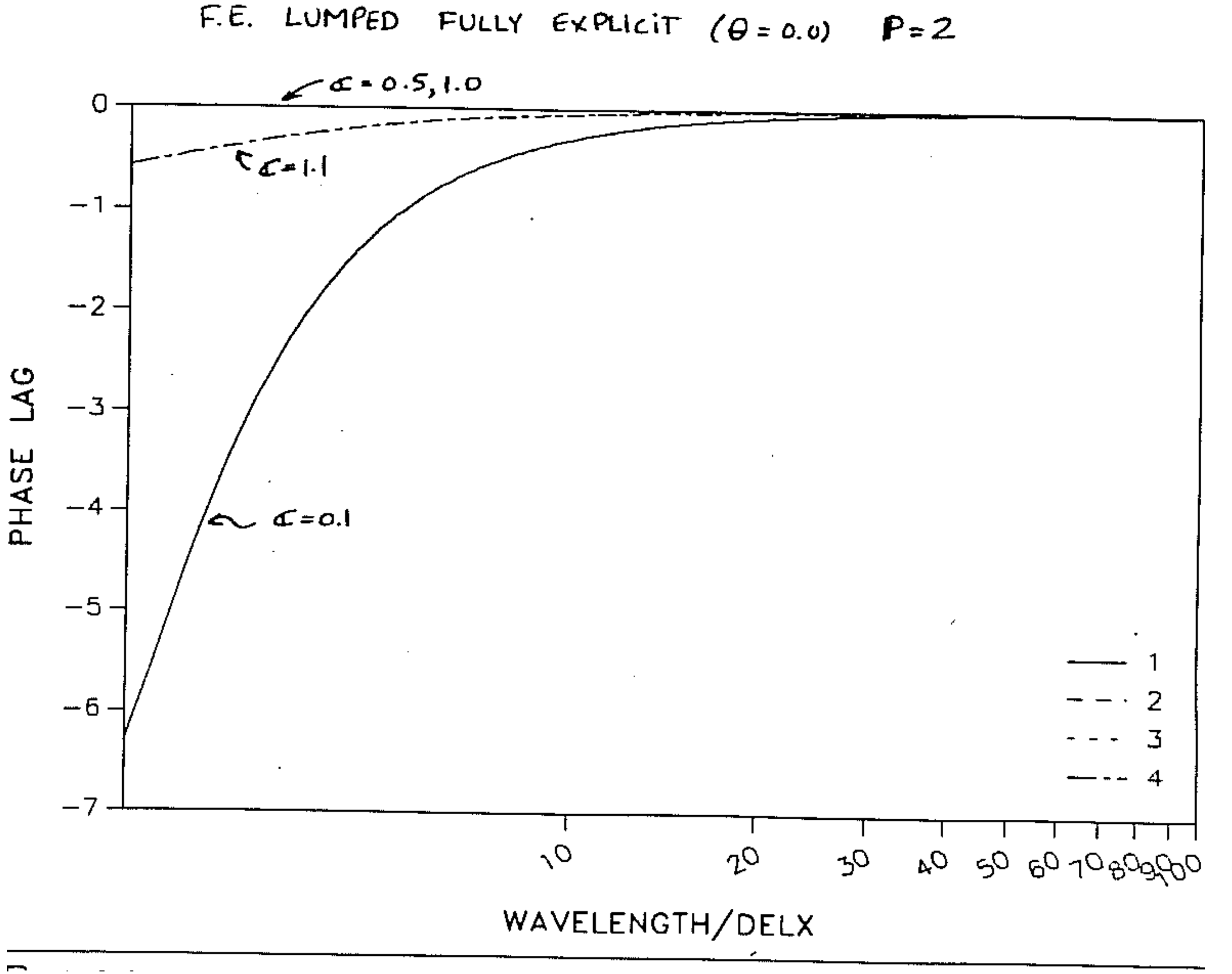

F.E. CONSISTENT FULLY EXPLICIT (θ=0.0) P=2
RATIO OF AMPLIFICATION FACTORS
WAVELENGTH/DELX
1
2
3
4
C=1.0
C=1.1
C=0.5
C=0.1
0
1
2
3
4
5
6
10
20
30
40
50
60
70
80
90
100

Fig. L29.3c1



F.E. LUMPED FULLY EXPLICIT (θ = 0.0) P=2
C = 0.5, 1.0
C=1.1
C=0.1
0
−1
−2
−3
−4
−5
−6
−7
PHASE LAG
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX
1
2
3
4

Fig. L29.3c2

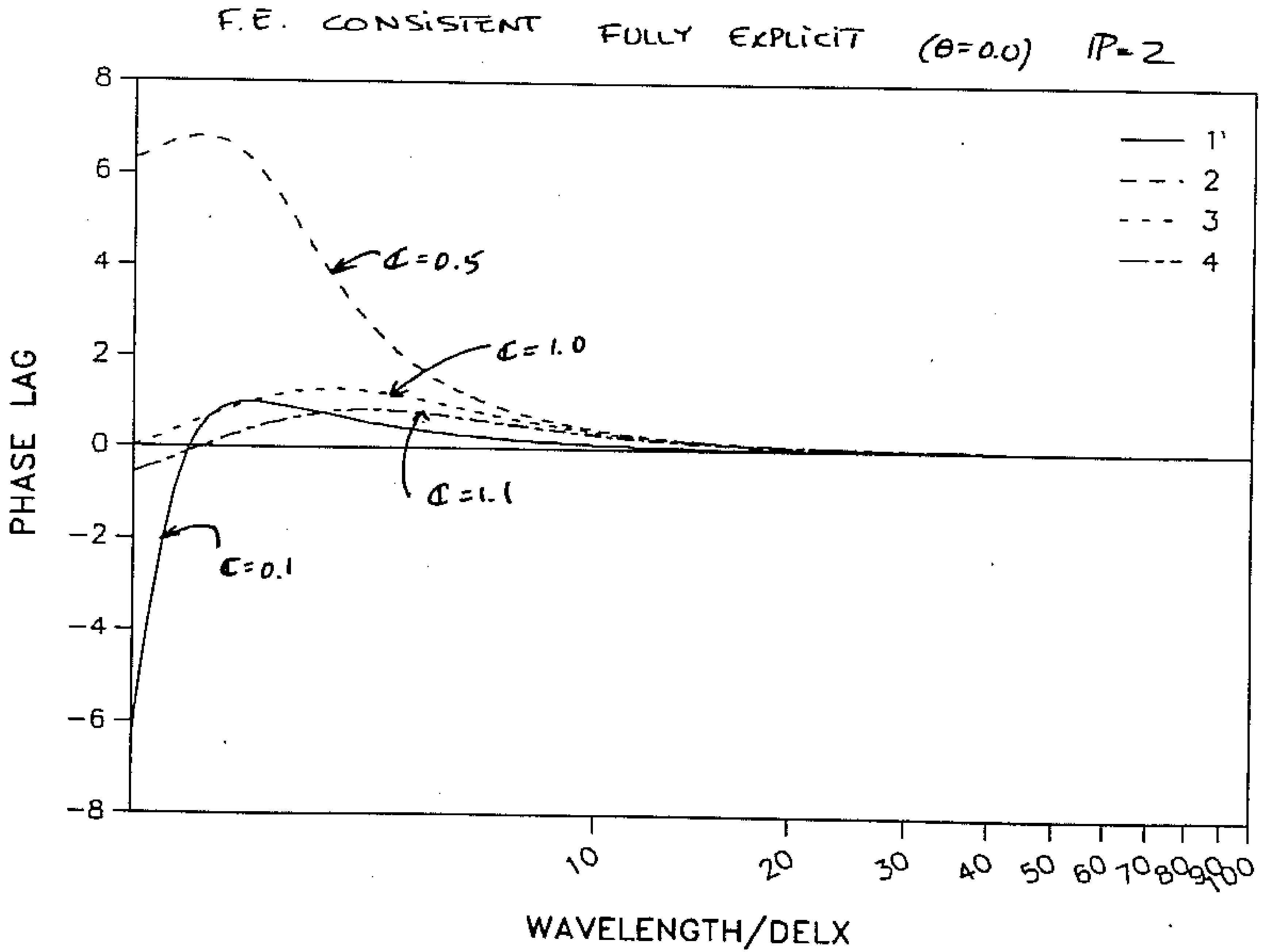

F.E. CONSISTENT FULLY EXPLICIT (θ=0.0) IP=2
PHASE LAG
WAVELENGTH/DELX
8
6
4
2
0
−2
−4
−6
−8
10
20
30
40
50
60
70
80
90
100
1
2
3
4
C=0.5
C=1.0
C=1.1
C=0.1

Fig. L29.4a1

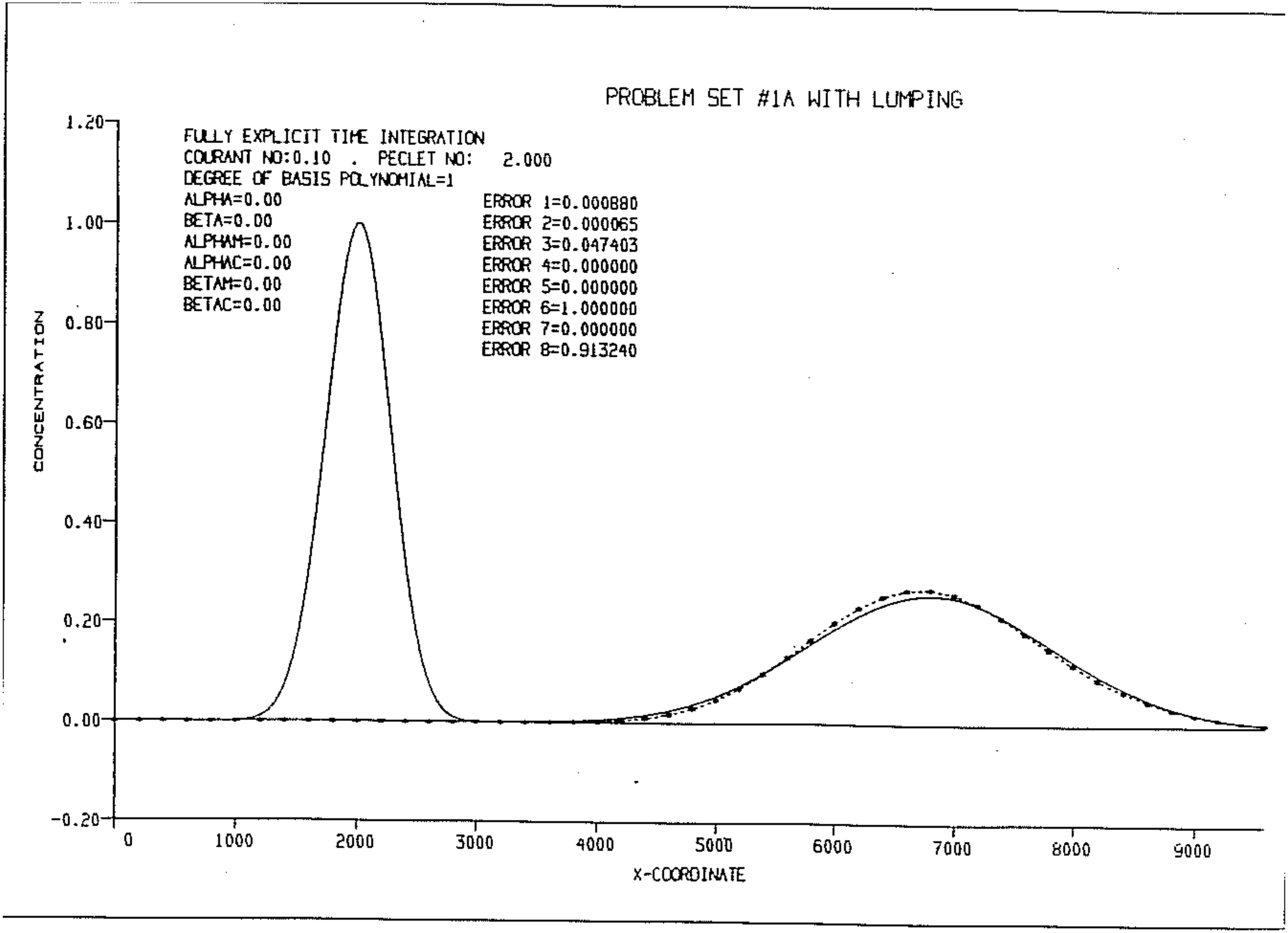
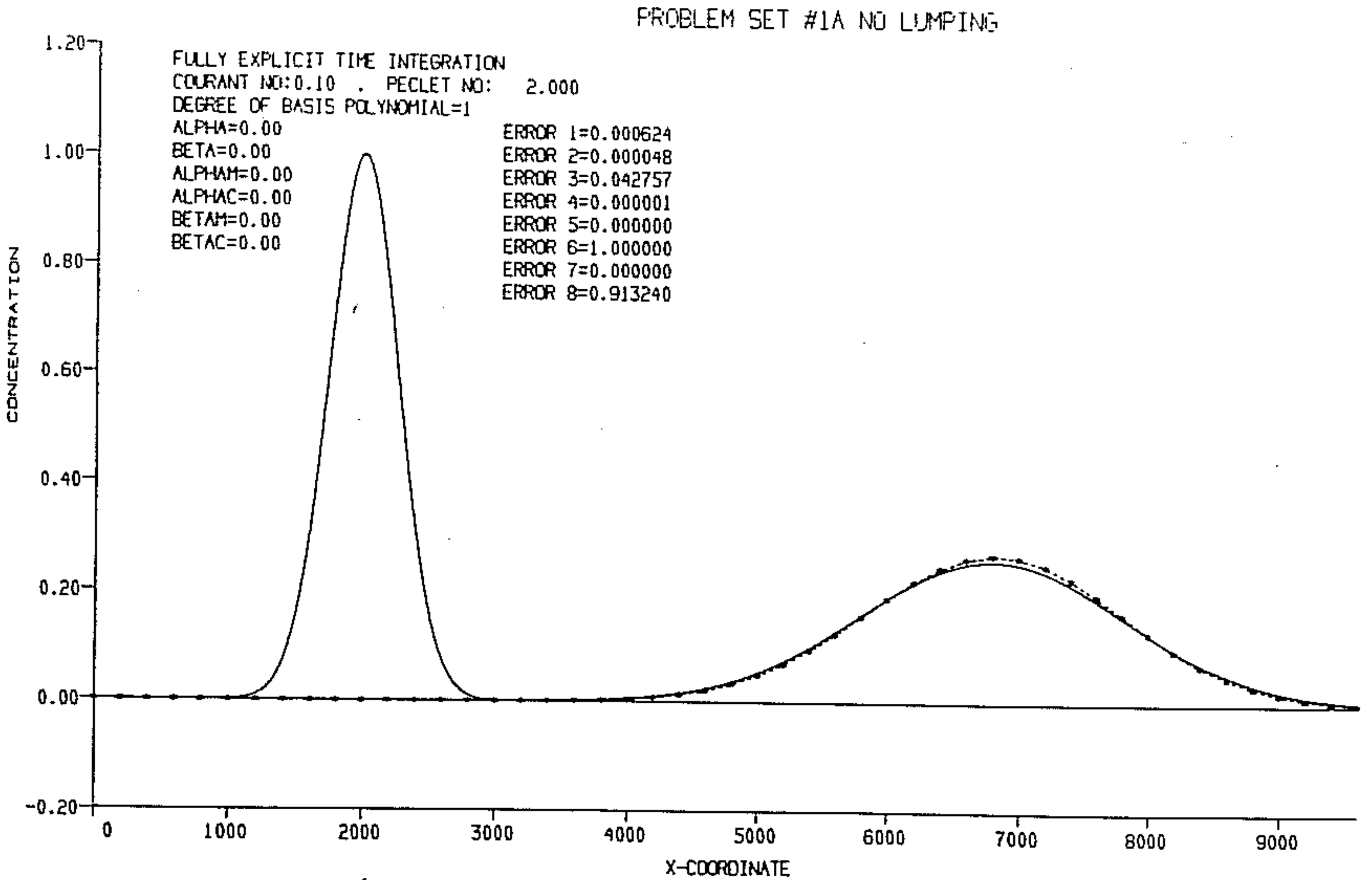

PROBLEM SET #1A WITH LUMPING
FULLY EXPLICIT TIME INTEGRATION
COURANT NO:0.10 . PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.000880
ERROR 2=0.000065
ERROR 3=0.047403
ERROR 4=0.000000
ERROR 5=0.000000
ERROR 6=1.000000
ERROR 7=0.000000
ERROR 8=0.913240
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.4a2



PROBLEM SET #1A NO LUMPING
FULLY EXPLICIT TIME INTEGRATION
COURANT NO:0.10 . PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.000624
ERROR 2=0.000048
ERROR 3=0.042757
ERROR 4=0.000001
ERROR 5=0.000000
ERROR 6=1.000000
ERROR 7=0.000000
ERROR 8=0.913240
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.4b1

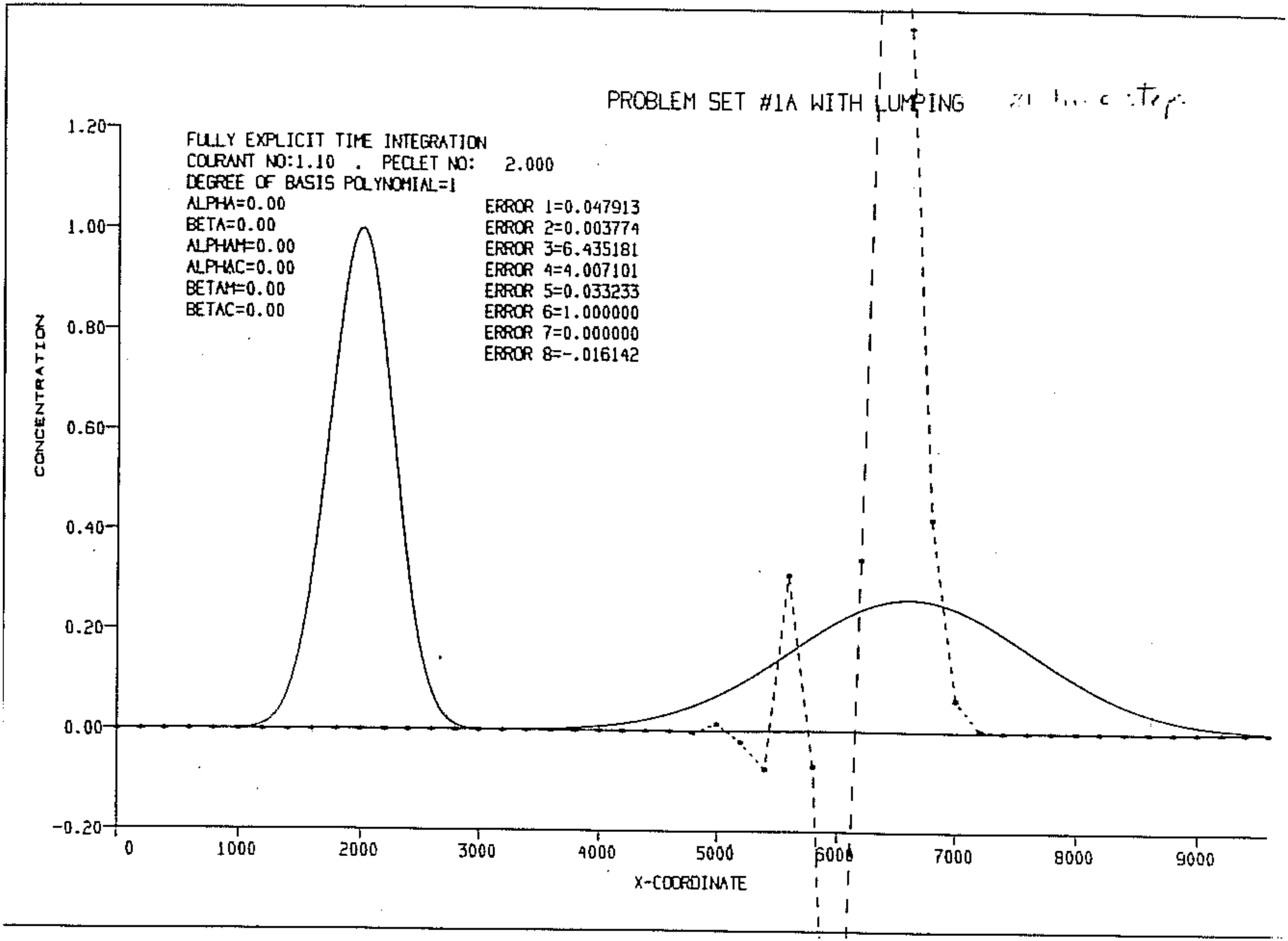

PROBLEM SET #1A WITH LUMPING
FULLY EXPLICIT TIME INTEGRATION
COURANT NO:1.10 . PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.047913
ERROR 2=0.003774
ERROR 3=6.435181
ERROR 4=4.007101
ERROR 5=0.033233
ERROR 6=1.000000
ERROR 7=0.000000
ERROR 8=-.016142
CONCENTRATION
X-COORDINATE
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000

Fig. L29.4b2

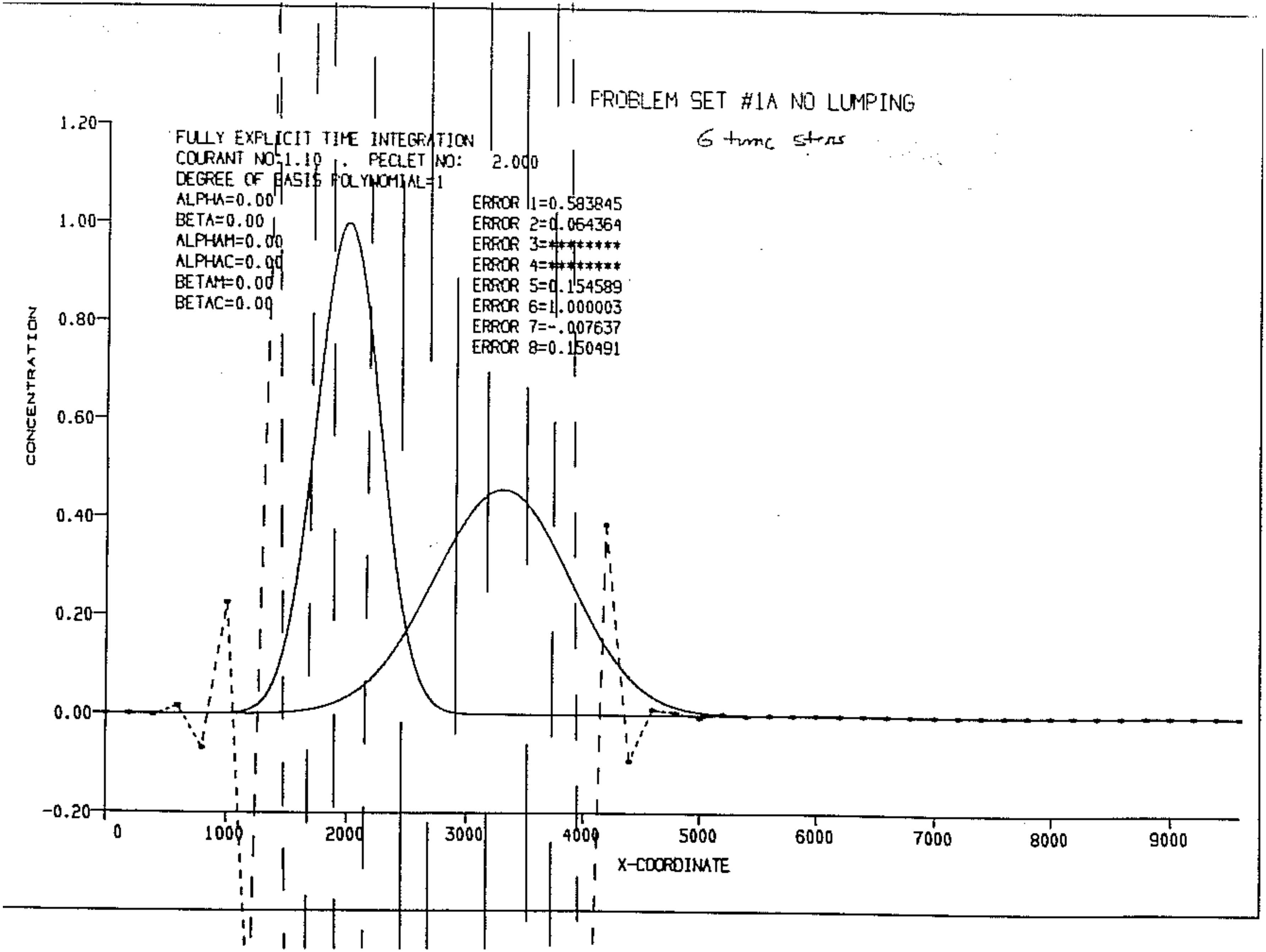
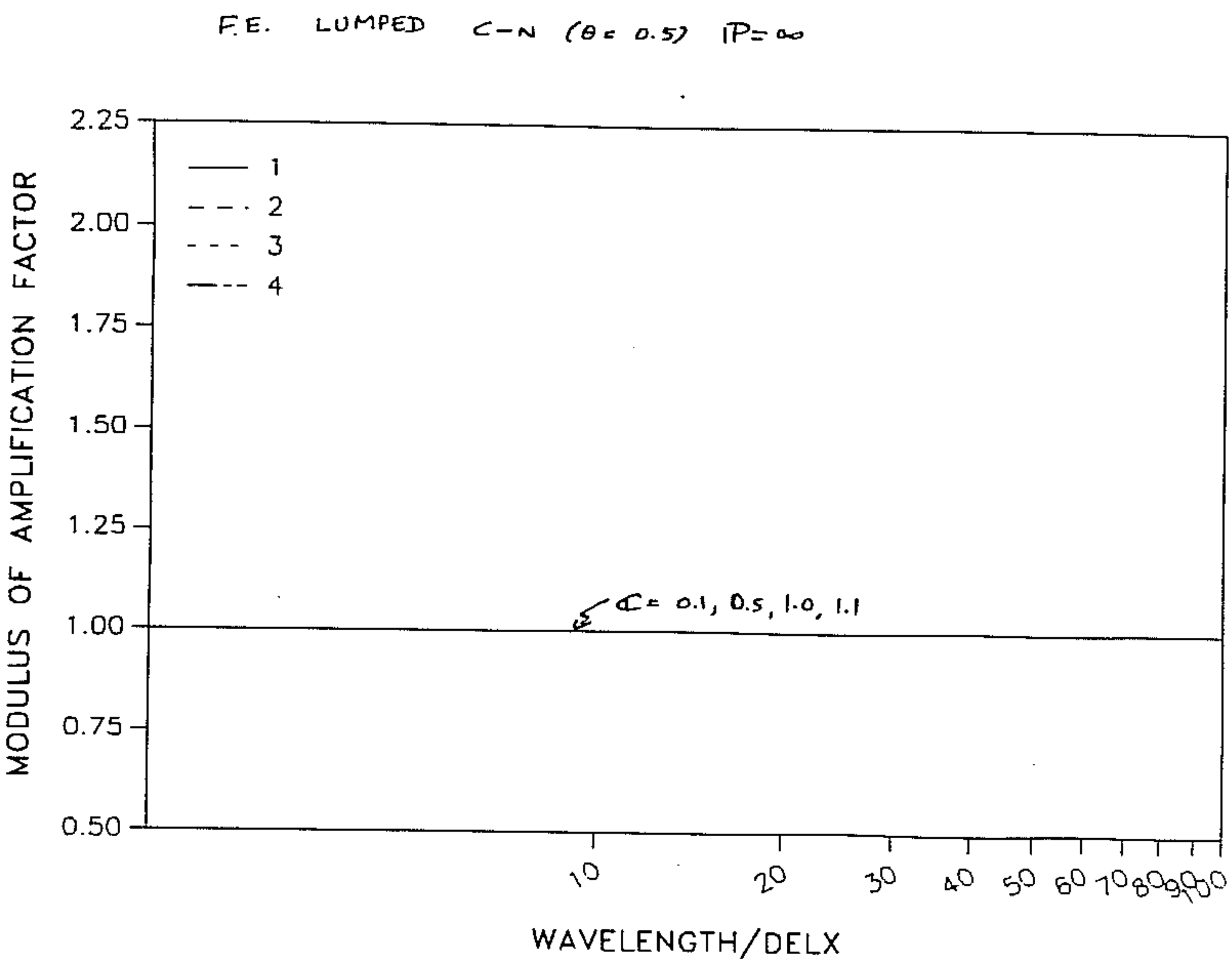

PROBLEM SET #1A NO LUMPING
6 time steps
FULLY EXPLICIT TIME INTEGRATION
COURANT NO:1.10 . PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.583845
ERROR 2=0.064364
ERROR 3=********
ERROR 4=********
ERROR 5=0.154589
ERROR 6=1.000003
ERROR 7=-.007637
ERROR 8=0.150491
CONCENTRATION
X-COORDINATE
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000

Fig. L29.5a1



F.E. LUMPED C-N (θ = 0.5) IP = ∞
1
2
3
4
C = 0.1, 0.5, 1.0, 1.1
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
MODULUS OF AMPLIFICATION FACTOR
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.5a2

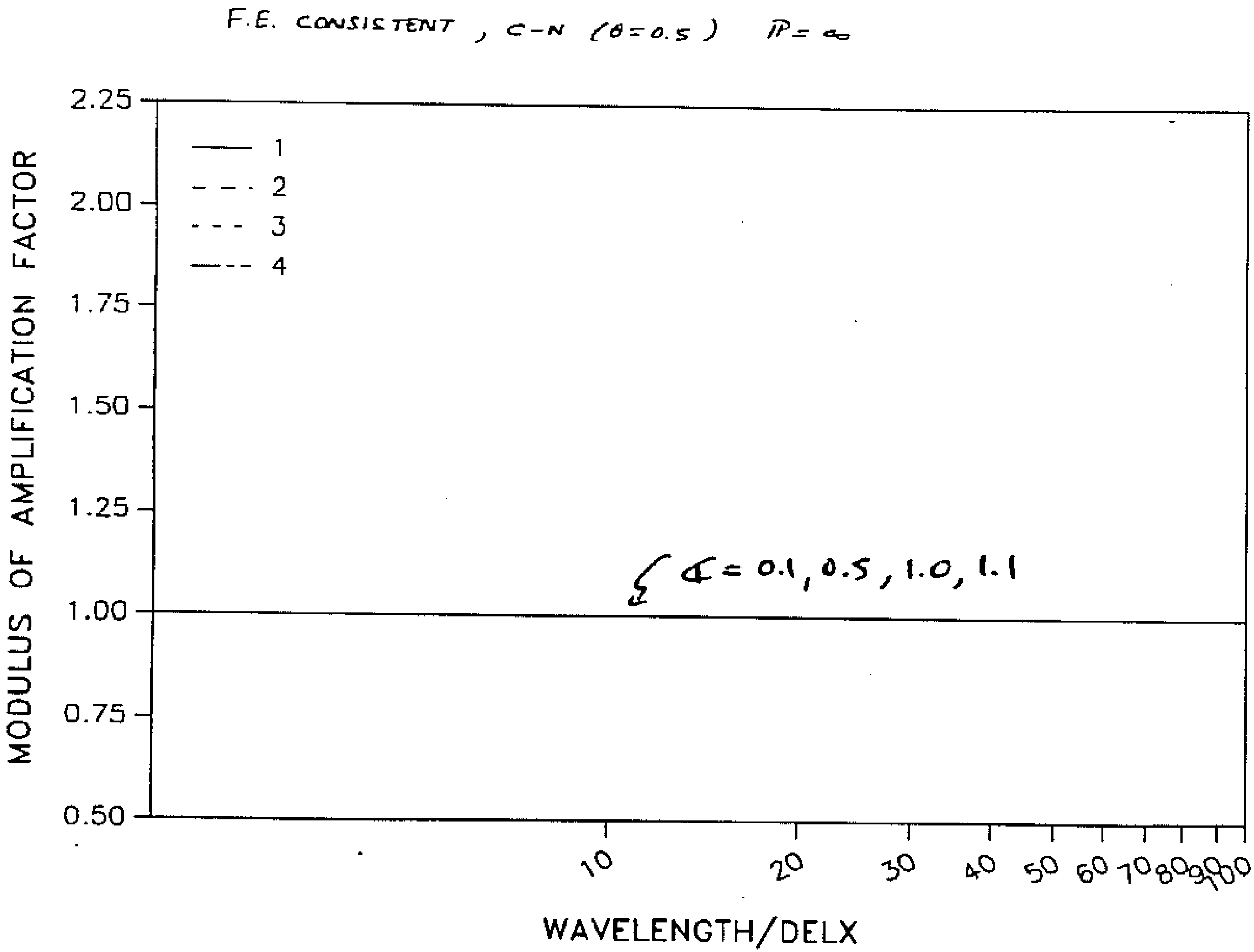
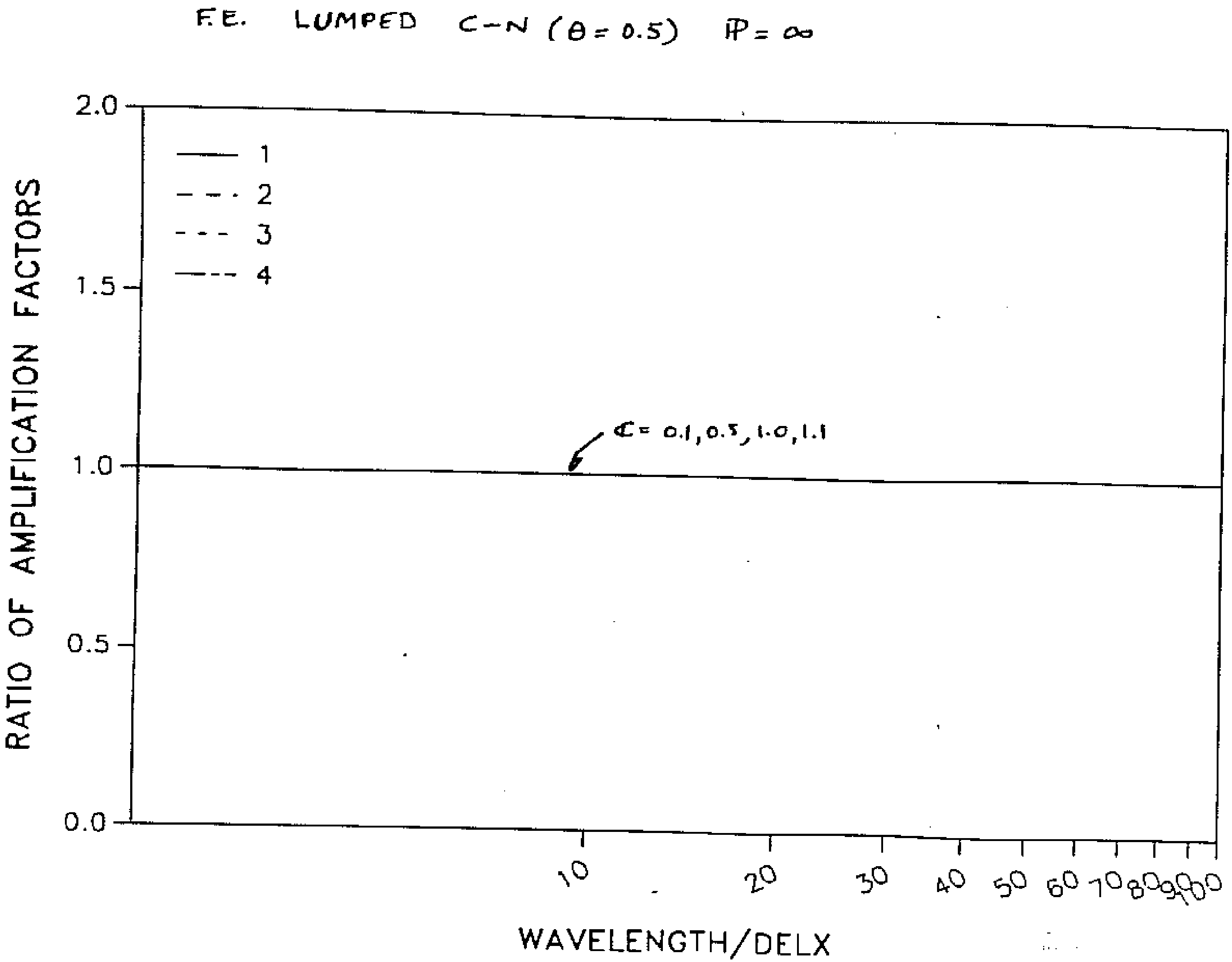

F.E. CONSISTENT , C-N (θ=0.5) IP= ∞
MODULUS OF AMPLIFICATION FACTOR
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
1
2
3
4
C = 0.1, 0.5, 1.0, 1.1
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.5b1



F.E. LUMPED C-N (θ = 0.5) IP = ∞
2.0
1.5
1.0
0.5
0.0
1
2
3
4
C = 0.1, 0.5, 1.0, 1.1
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX
RATIO OF AMPLIFICATION FACTORS

Fig. L29.5b2

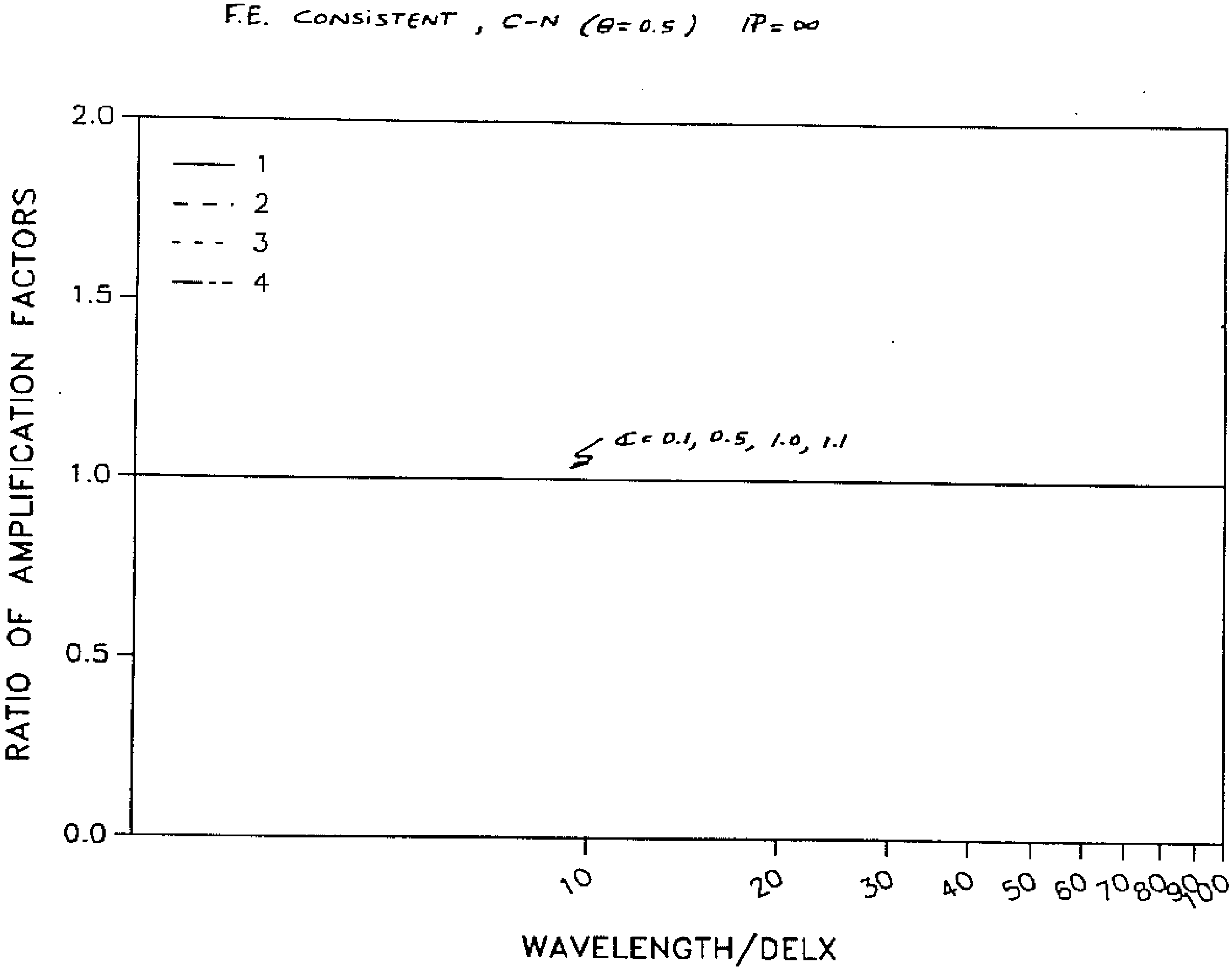

F.E. CONSISTENT , C-N (θ= 0.5) IP = ∞
RATIO OF AMPLIFICATION FACTORS
2.0
1.5
1.0
0.5
0.0
1
2
3
4
C = 0.1, 0.5, 1.0, 1.1
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.5c1

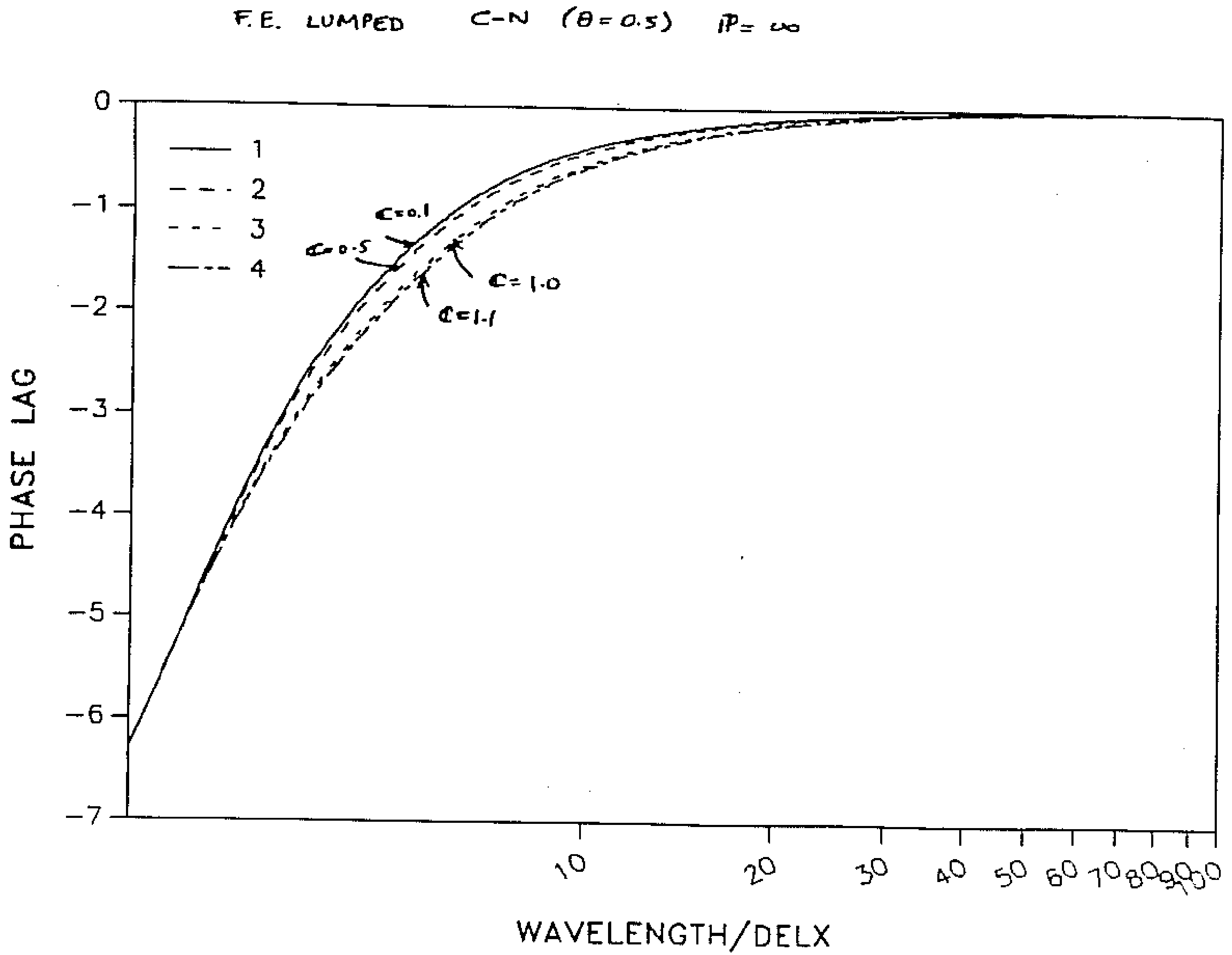

F.E. LUMPED C-N (θ = 0.5) P = ∞
0
−1
−2
−3
−4
−5
−6
−7
PHASE LAG
1
2
3
4
C=0.1
C=0.5
C=1.0
C=1.1
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.5c2

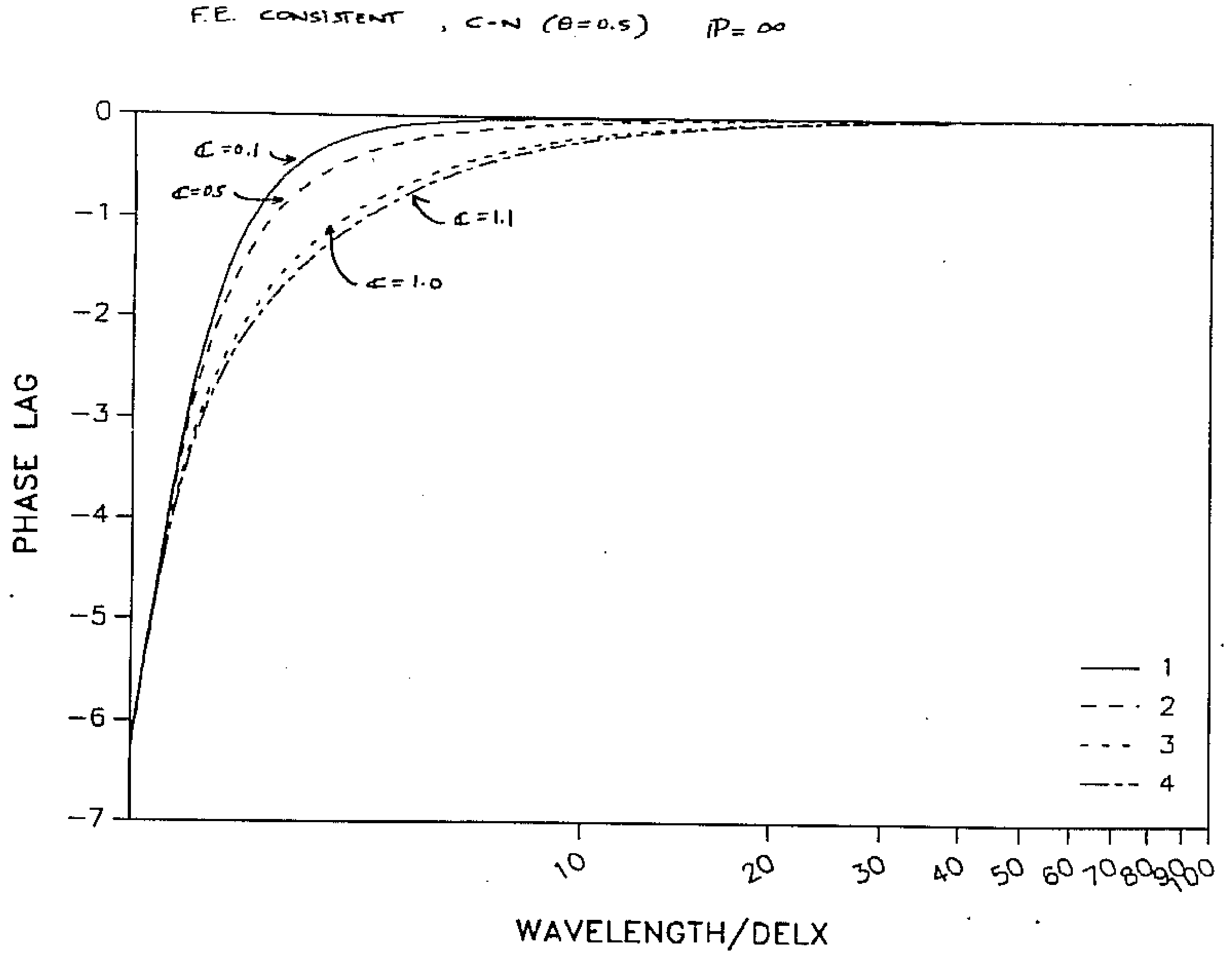

F.E. CONSISTENT , C-N (θ=0.5) IP= ∞
0
−1
−2
−3
−4
−5
−6
−7
PHASE LAG
C=0.1
C=0.5
C=1.1
C=1.0
1
2
3
4
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.6a1

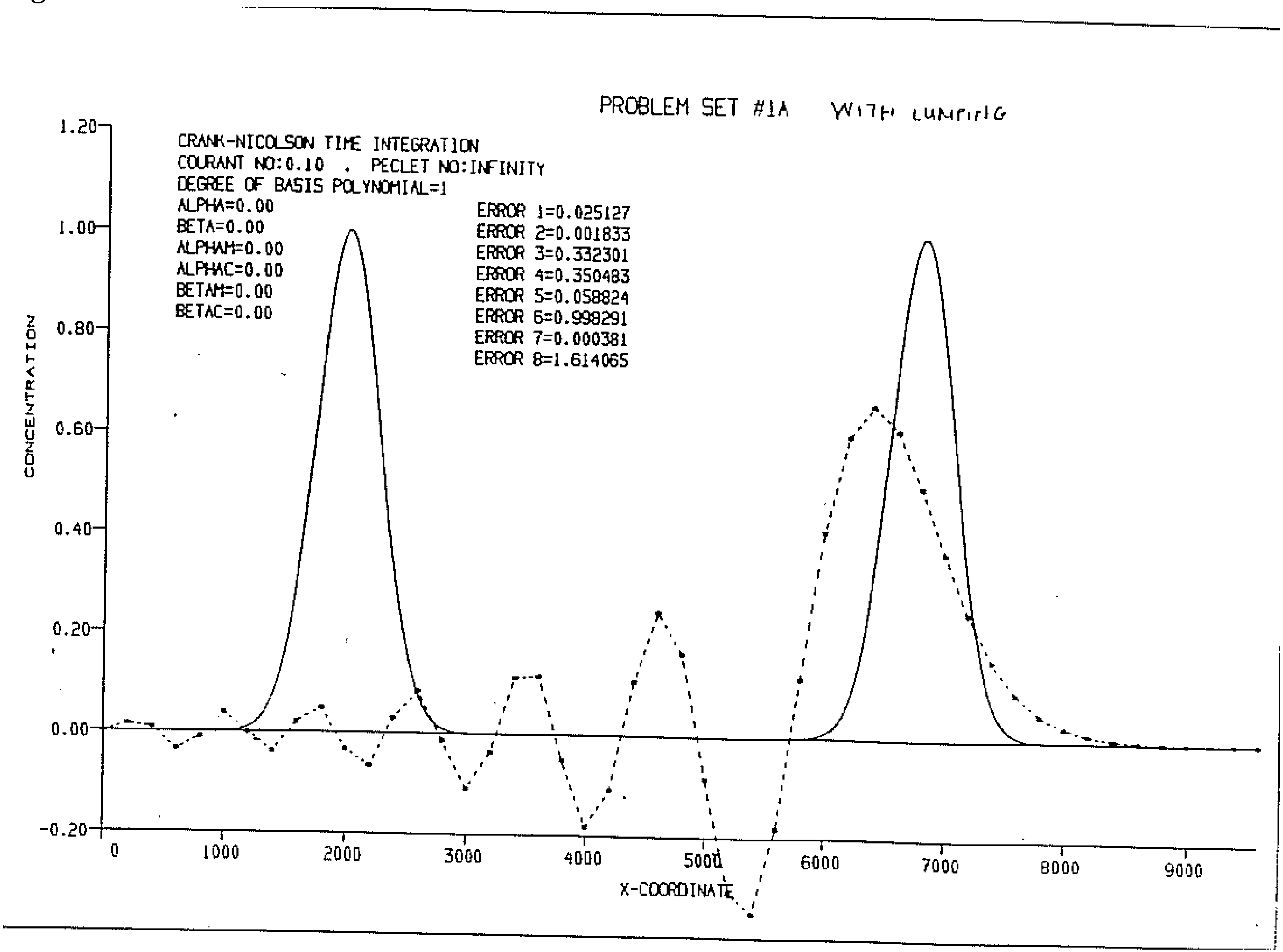
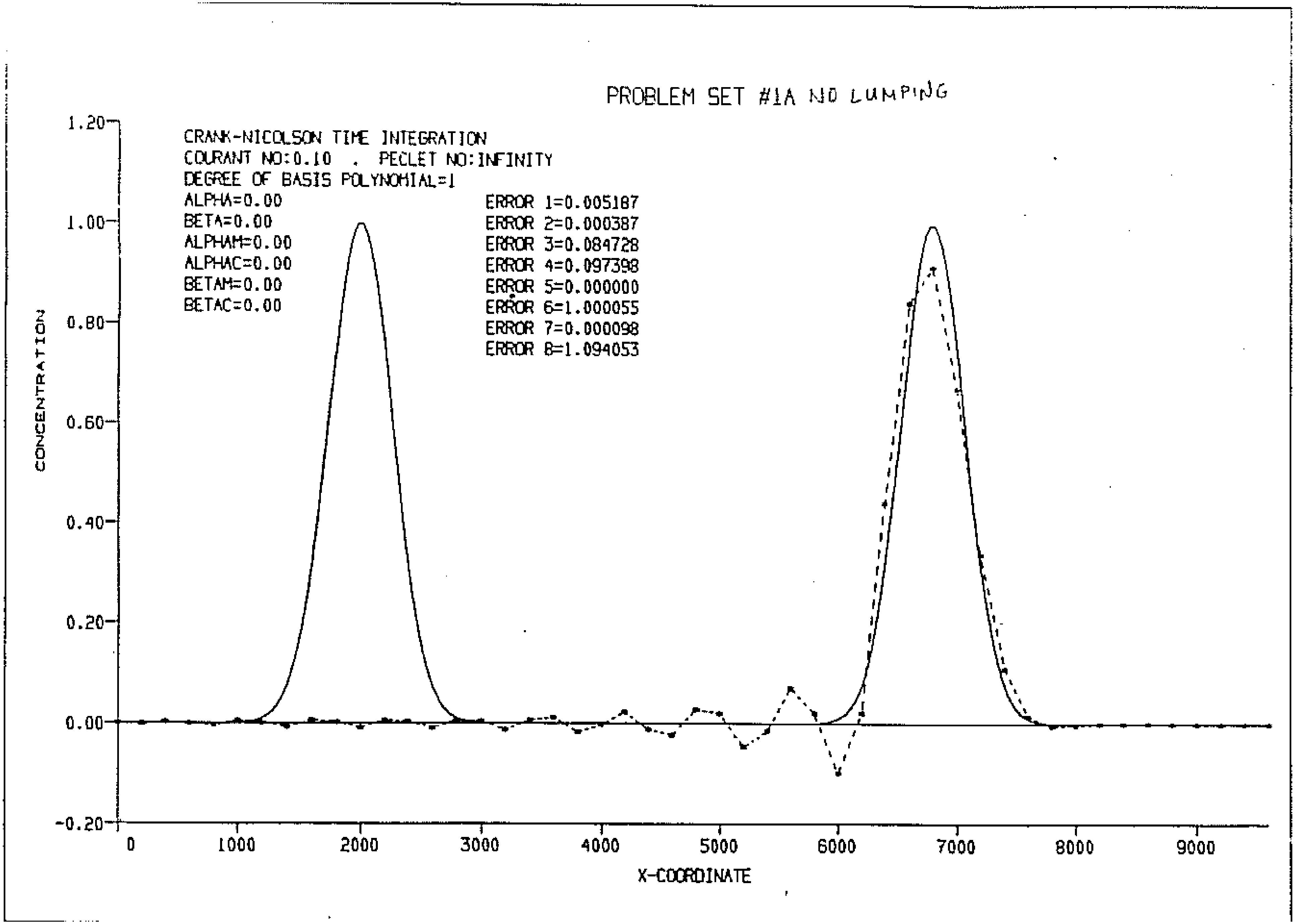
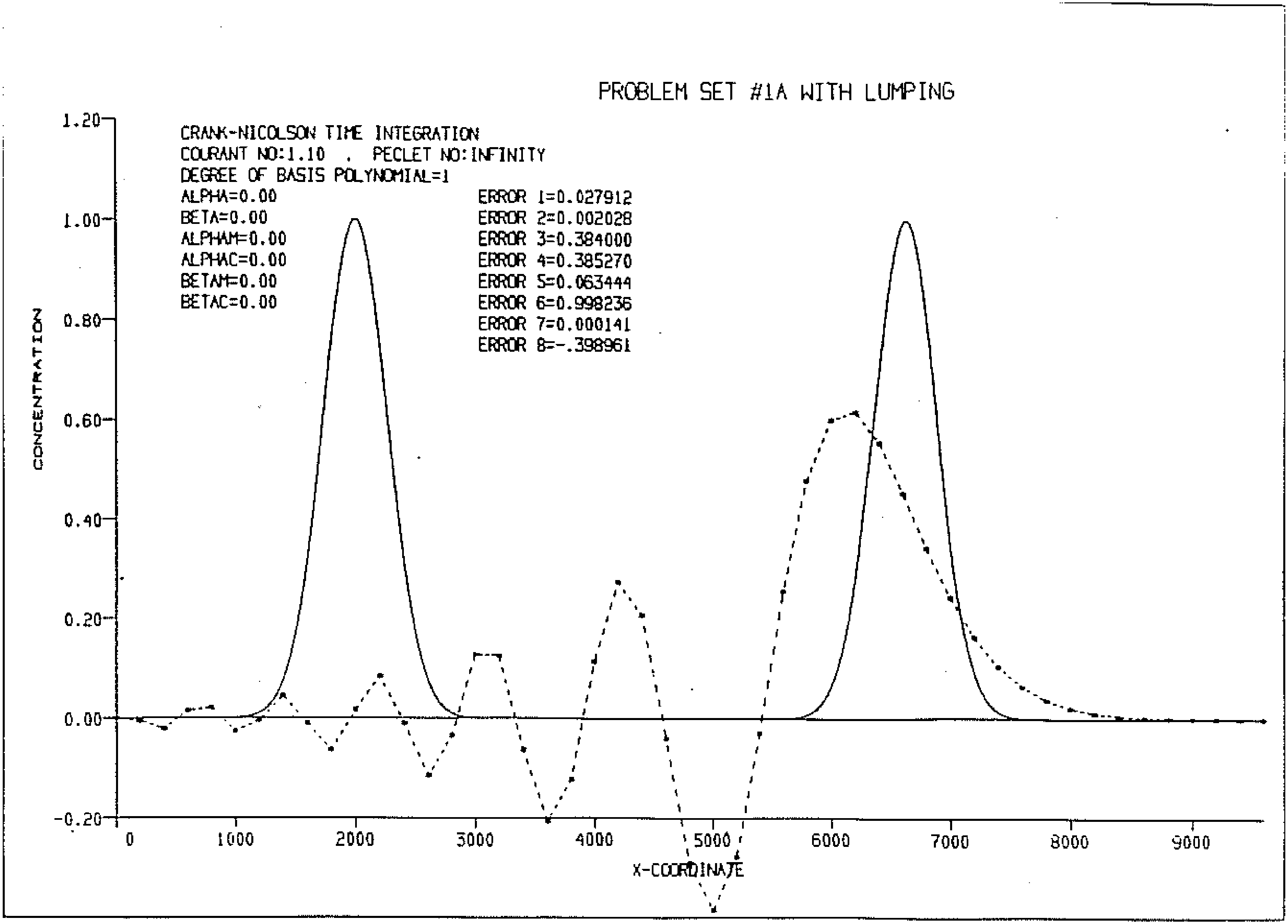

PROBLEM SET #1A WITH LUMPING
CRANK-NICOLSON TIME INTEGRATION
COURANT NO:0.10 , PECLET NO:INFINITY
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.025127
ERROR 2=0.001833
ERROR 3=0.332301
ERROR 4=0.350483
ERROR 5=0.058824
ERROR 6=0.998291
ERROR 7=0.000381
ERROR 8=1.614065
CONCENTRATION
X-COORDINATE
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000

Fig. L29.6a2



PROBLEM SET #1A NO LUMPING
CRANK-NICOLSON TIME INTEGRATION
COURANT NO:0.10 . PECLET NO:INFINITY
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.005187
ERROR 2=0.000387
ERROR 3=0.084728
ERROR 4=0.097398
ERROR 5=0.000000
ERROR 6=1.000055
ERROR 7=0.000098
ERROR 8=1.094053
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.6b1



PROBLEM SET #1A WITH LUMPING
CRANK-NICOLSON TIME INTEGRATION
COURANT NO:1.10 , PECLET NO:INFINITY
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.027912
ERROR 2=0.002028
ERROR 3=0.384000
ERROR 4=0.385270
ERROR 5=0.063444
ERROR 6=0.998236
ERROR 7=0.000141
ERROR 8=-.398961
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.6b2

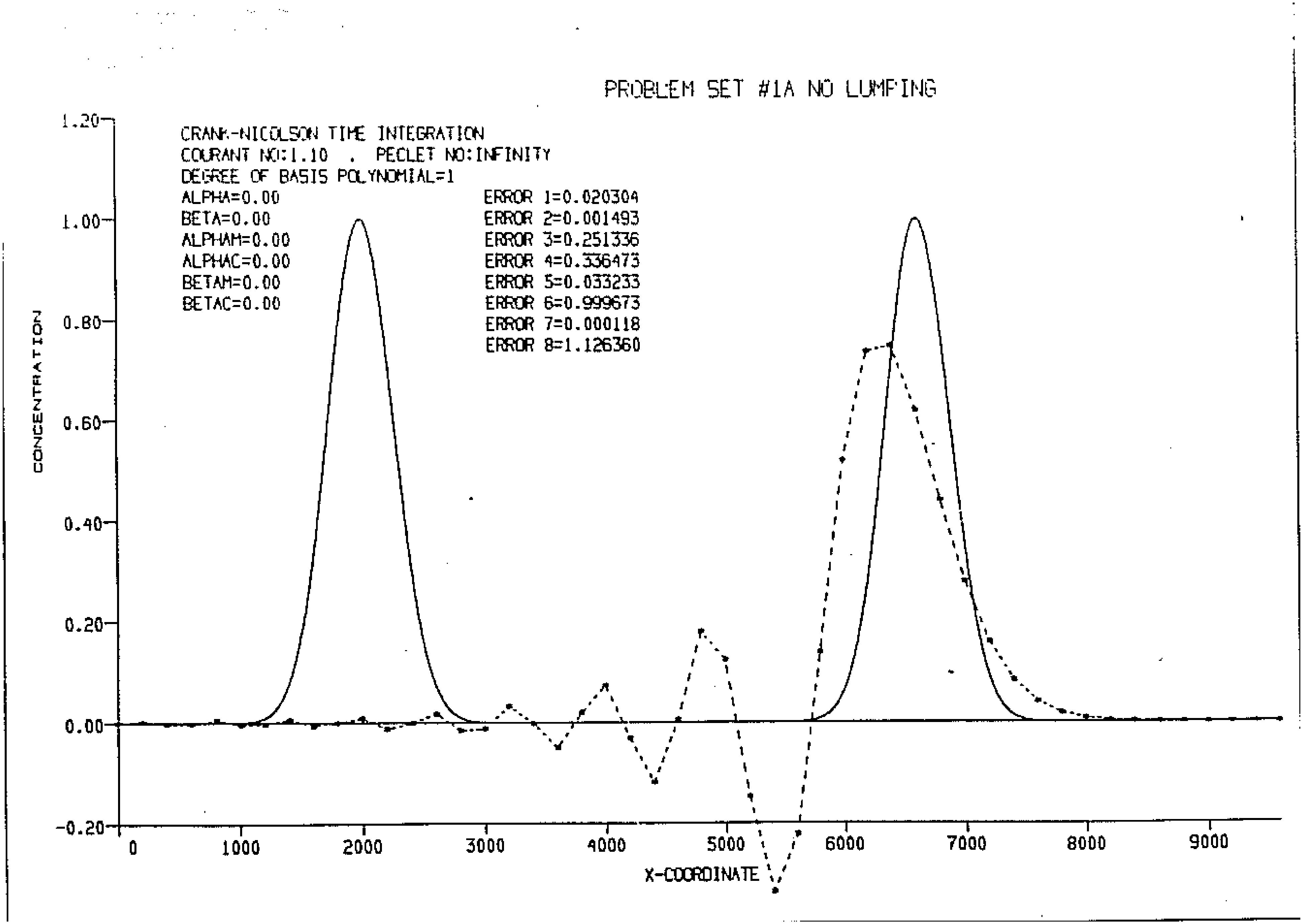

PROBLEM SET #1A NO LUMPING
CRANK-NICOLSON TIME INTEGRATION
COURANT NO:1.10 . PECLET NO:INFINITY
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.020304
ERROR 2=0.001493
ERROR 3=0.251336
ERROR 4=0.336473
ERROR 5=0.033233
ERROR 6=0.999673
ERROR 7=0.000118
ERROR 8=1.126360
CONCENTRATION
X-COORDINATE
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000

Fig. L29.7a1

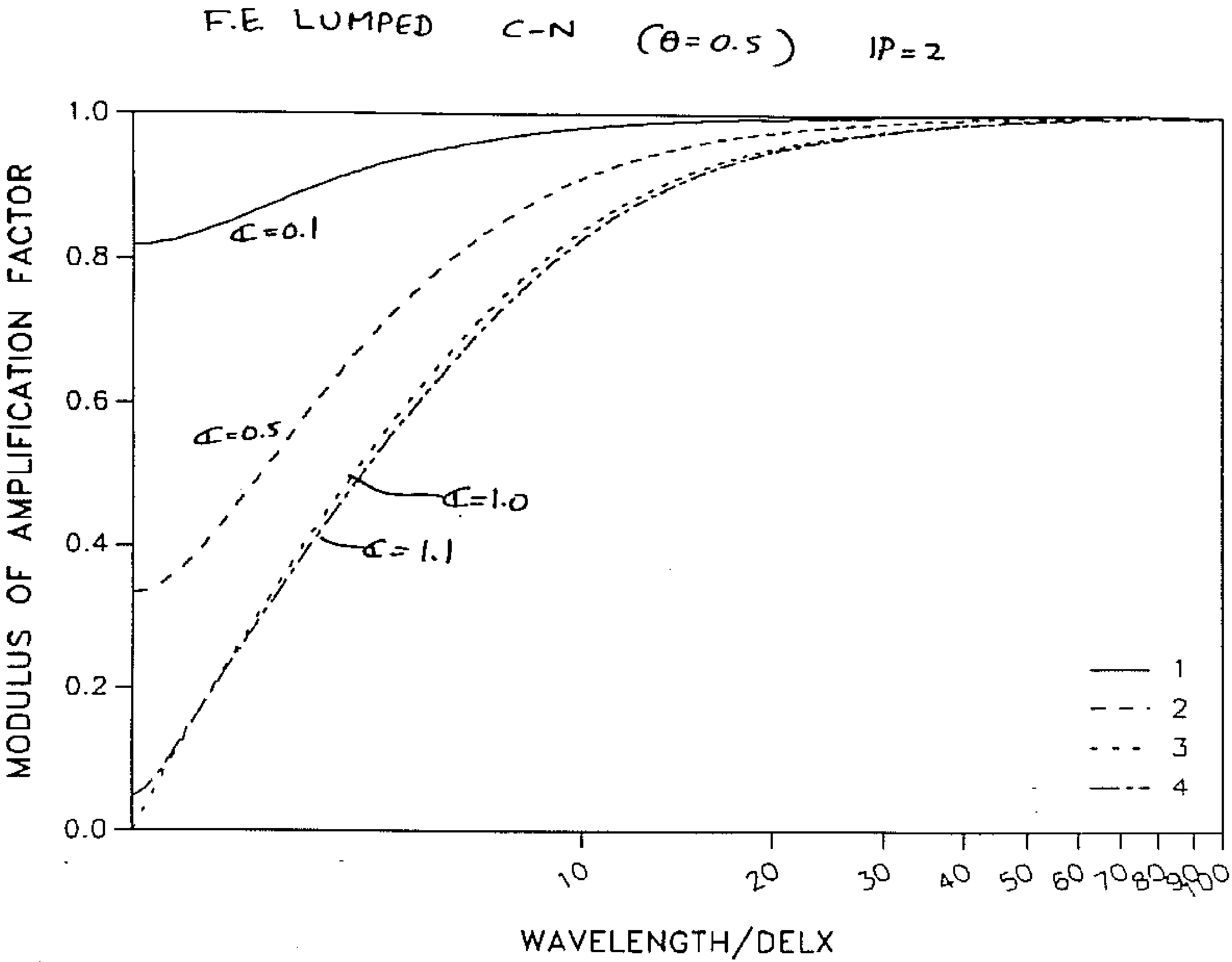
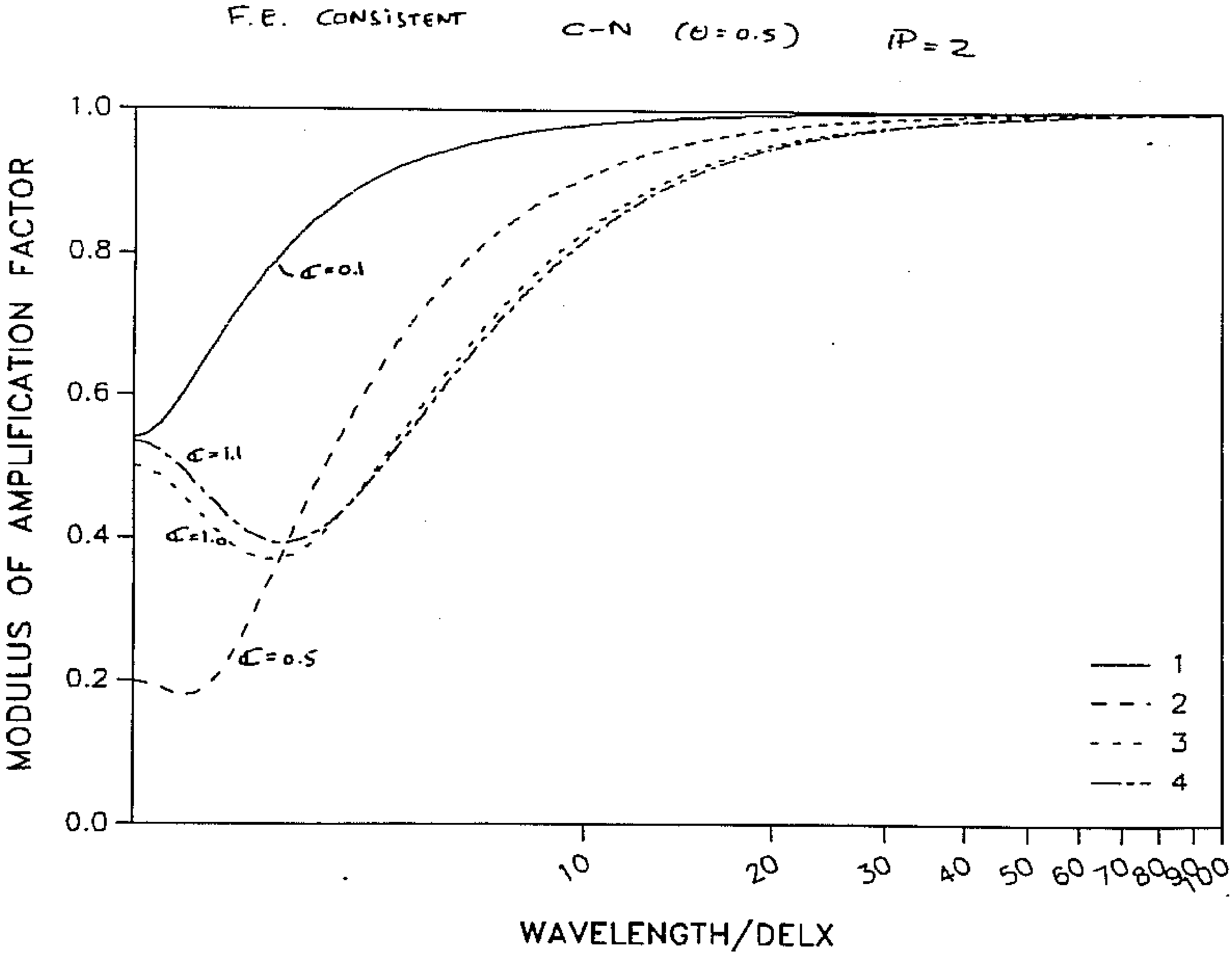

F.E LUMPED C-N (θ=0.5) IP=2
MODULUS OF AMPLIFICATION FACTOR
1.0
0.8
0.6
0.4
0.2
0.0
C=0.1
C=0.5
C=1.0
C=1.1
1
2
3
4
10
20
30
40
50
60
70
WAVELENGTH/DELX

Fig. L29.7a2



F.E. CONSISTENT
C-N (θ=0.5)
IP=2
MODULUS OF AMPLIFICATION FACTOR
1.0
0.8
0.6
0.4
0.2
0.0
C=0.1
C=1.1
C=1.0
C=0.5
1
2
3
4
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.7b1

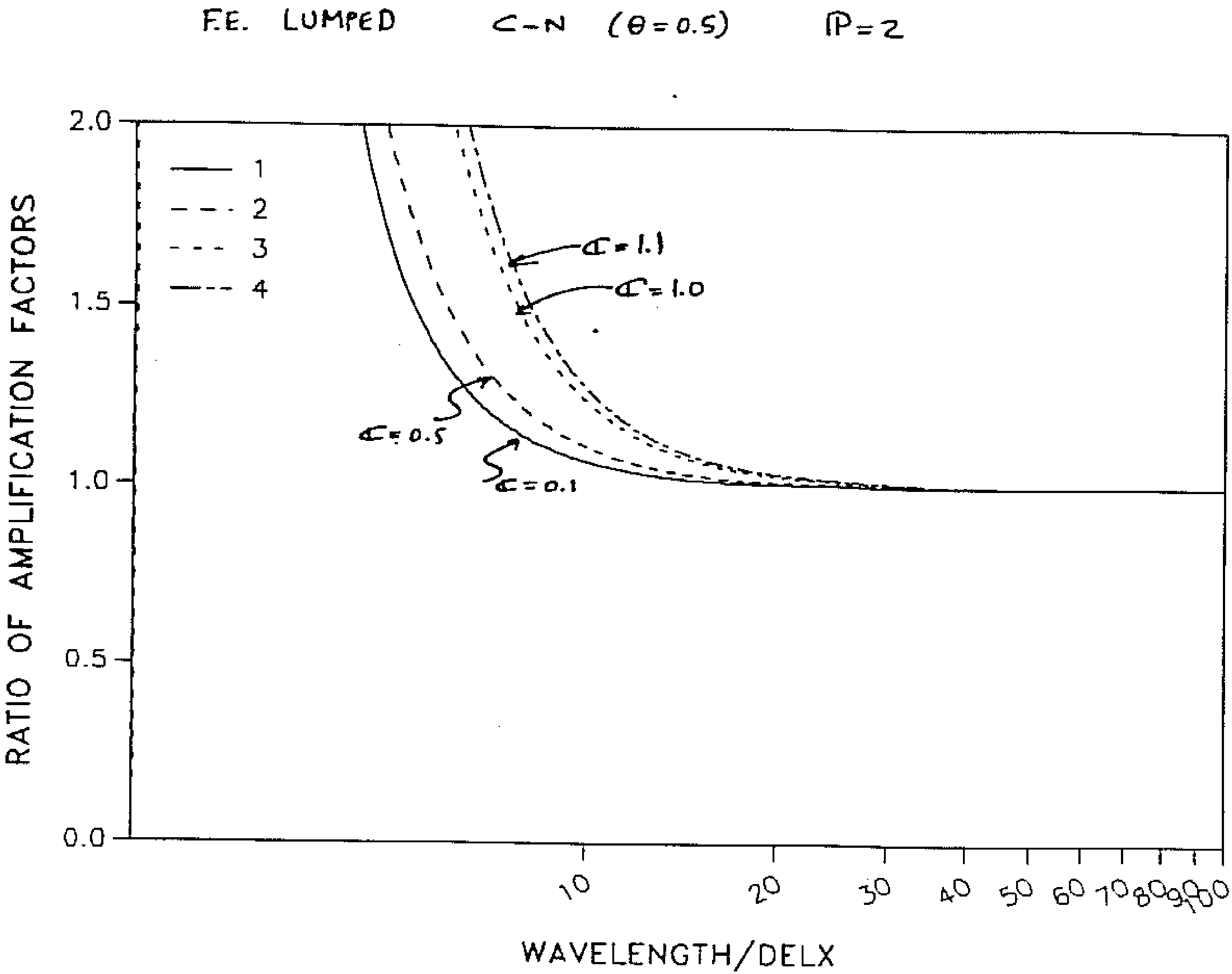

F.E. LUMPED C-N (θ=0.5) P=2
2.0
1.5
1.0
0.5
0.0
1
2
3
4
C=1.1
C=1.0
C=0.5
C=0.1
RATIO OF AMPLIFICATION FACTORS
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.7b2

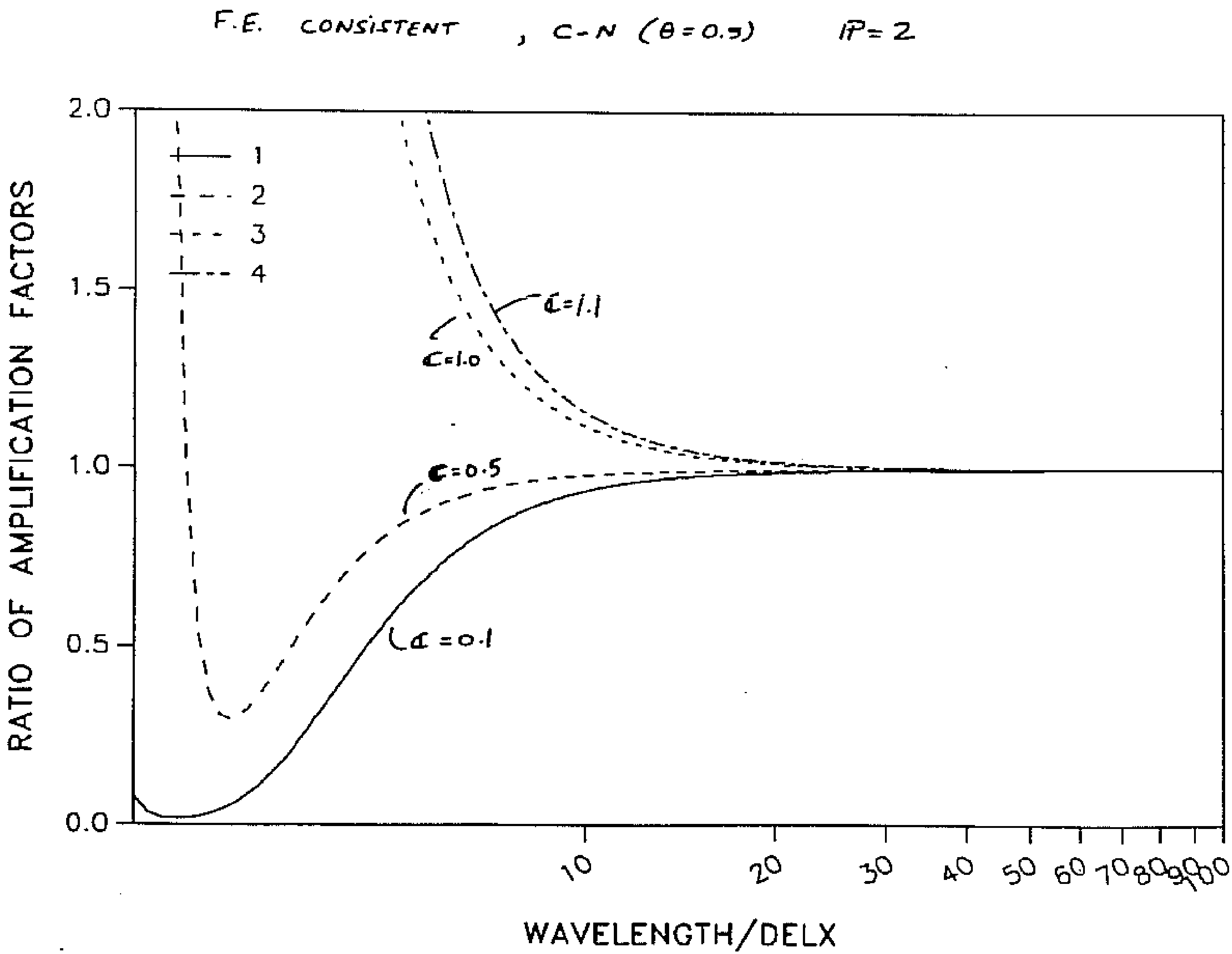
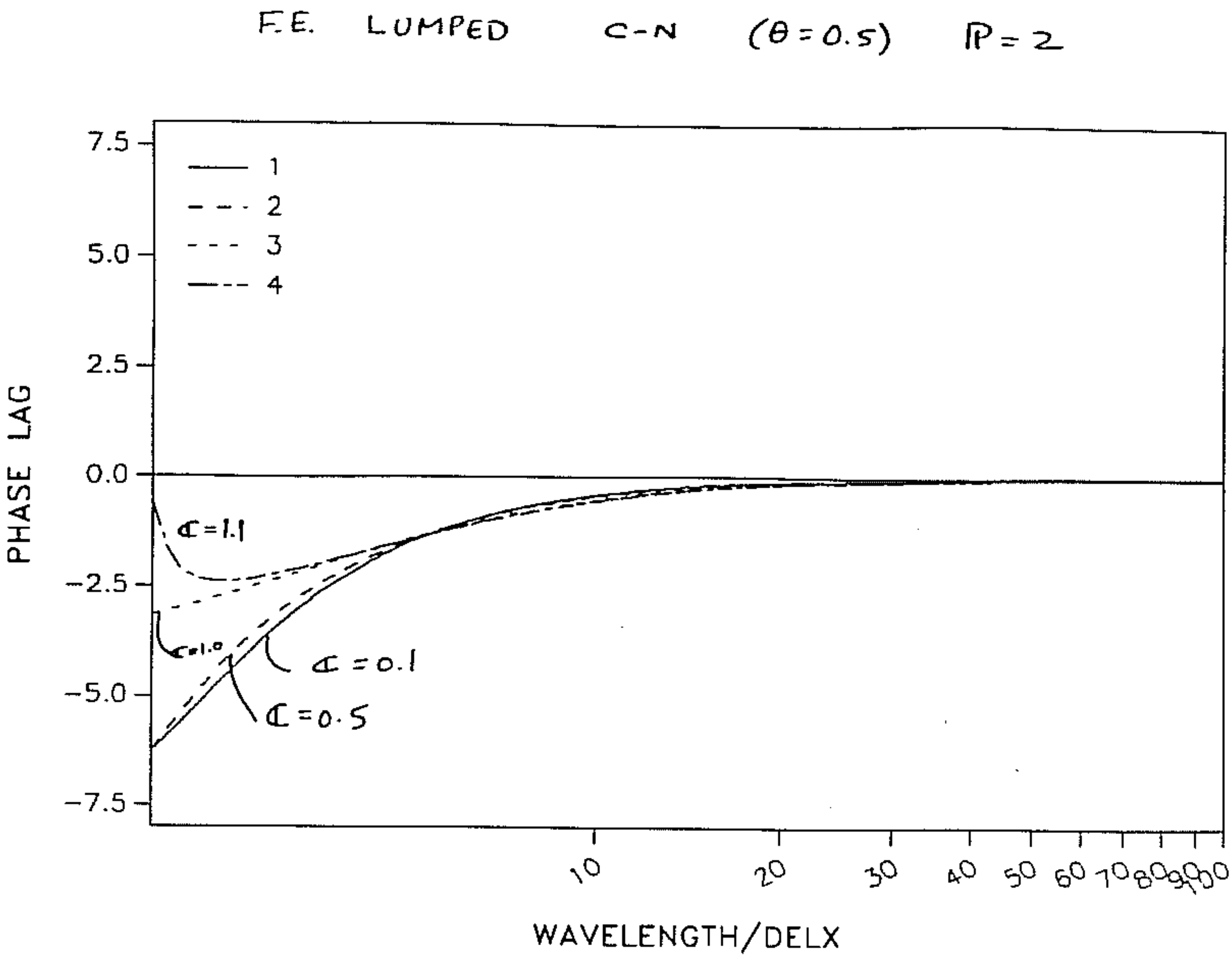

F.E. CONSISTENT , C-N (θ=0.5) IP=2
2.0
1.5
1.0
0.5
0.0
1
2
3
4
C=1.1
C=1.0
C=0.5
C=0.1
RATIO OF AMPLIFICATION FACTORS
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.7c1



F.E. LUMPED C-N (θ=0.5) ℙ=2
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
PHASE LAG
1
2
3
4
ℂ=1.1
ℂ=1.0
ℂ=0.1
ℂ=0.5
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.7c2

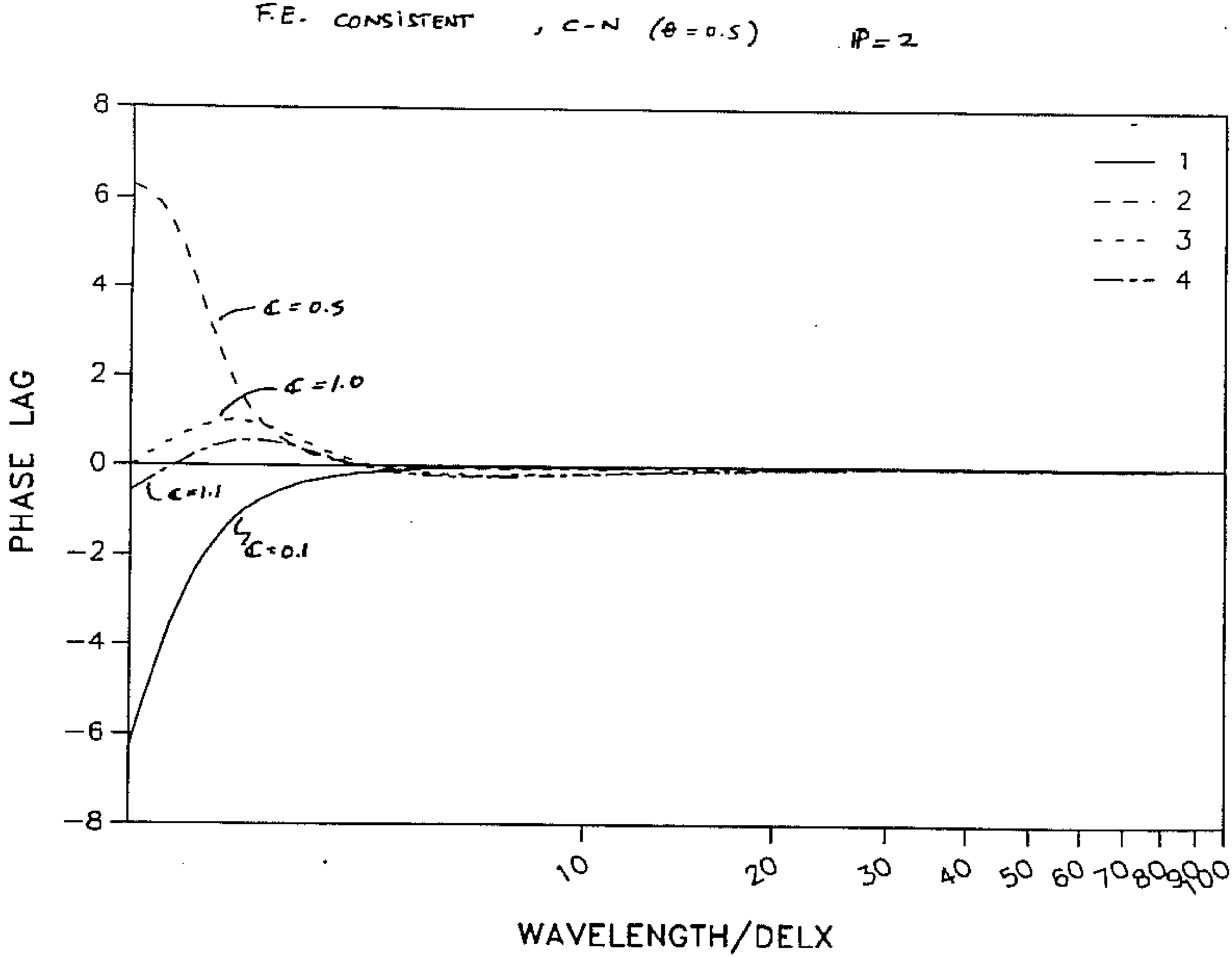

F.E. CONSISTENT , C-N (θ = 0.5) IP = 2
PHASE LAG
WAVELENGTH/DELX
8
6
4
2
0
−2
−4
−6
−8
10
20
30
40
50
60
70
80
90
100
1
2
3
4
C = 0.5
C = 1.0
C = 1.1
C = 0.1

Fig. L29.8a1

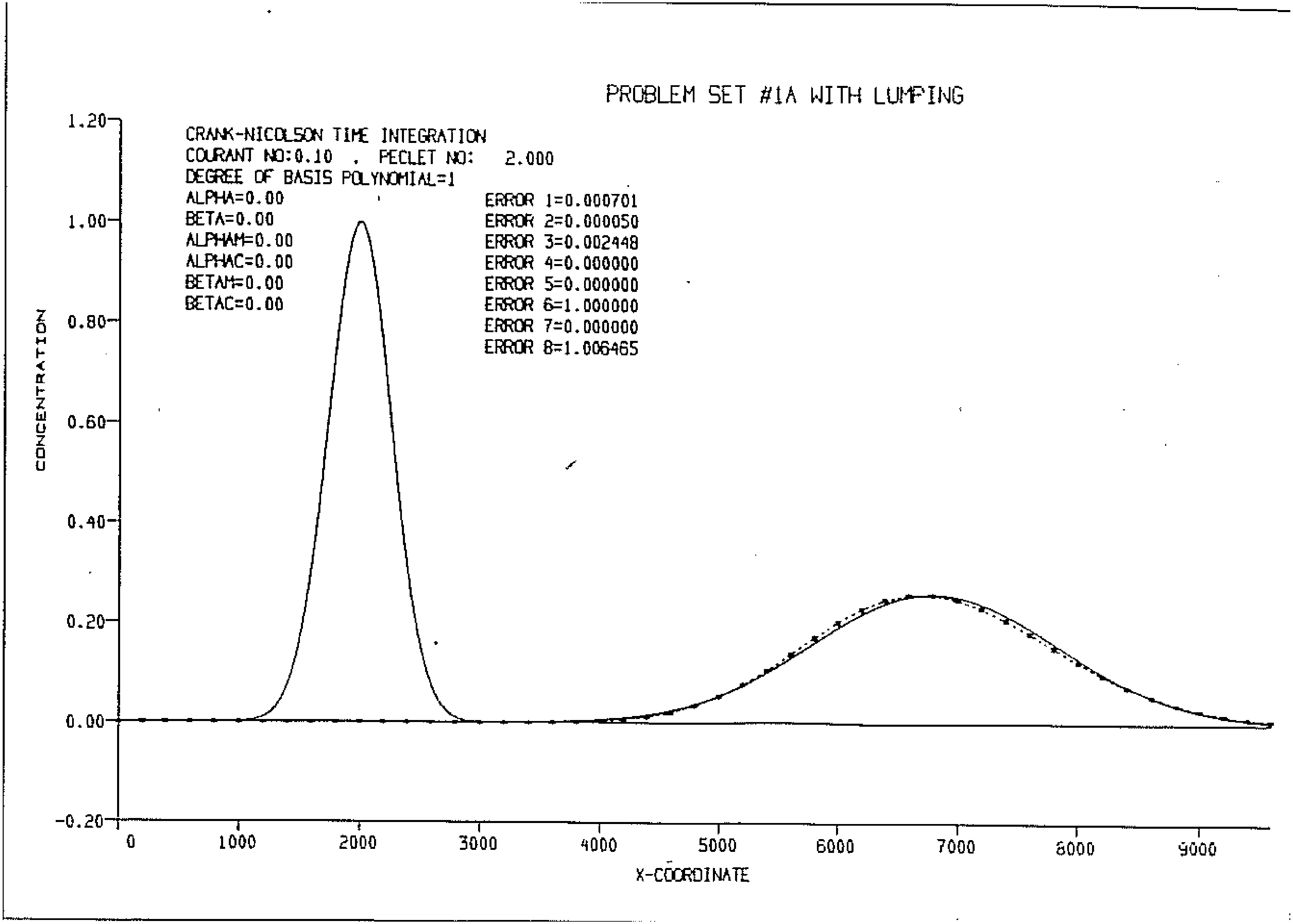

PROBLEM SET #1A WITH LUMPING
CRANK-NICOLSON TIME INTEGRATION
COURANT NO:0.10 , PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.000701
ERROR 2=0.000050
ERROR 3=0.002448
ERROR 4=0.000000
ERROR 5=0.000000
ERROR 6=1.000000
ERROR 7=0.000000
ERROR 8=1.006465
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.8a2

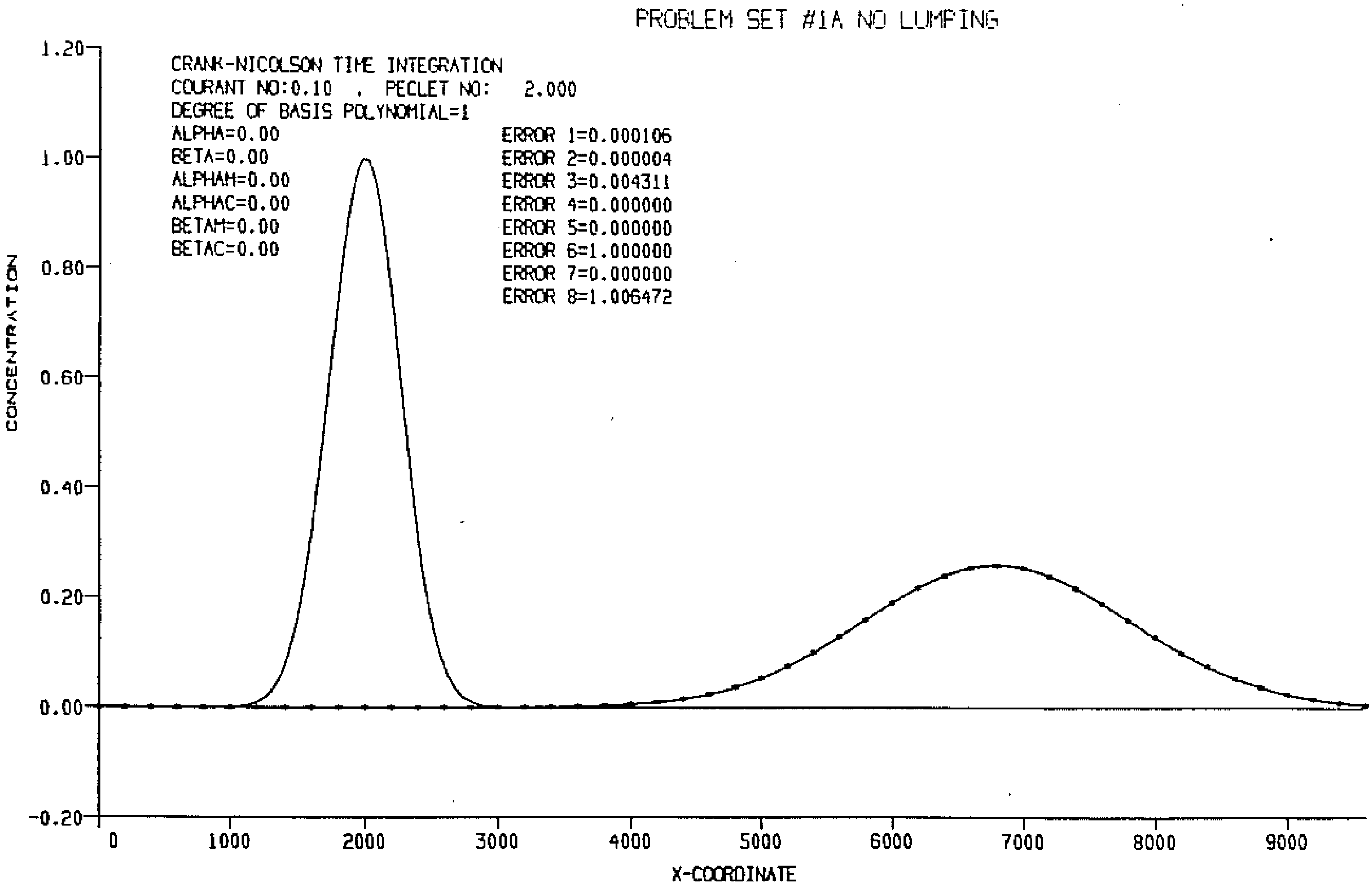

PROBLEM SET #1A NO LUMPING
CRANK-NICOLSON TIME INTEGRATION
COURANT NO:0.10 , PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.000106
ERROR 2=0.000004
ERROR 3=0.004311
ERROR 4=0.000000
ERROR 5=0.000000
ERROR 6=1.000000
ERROR 7=0.000000
ERROR 8=1.006472
CONCENTRATION
X-COORDINATE
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000

Fig. L29.8b1

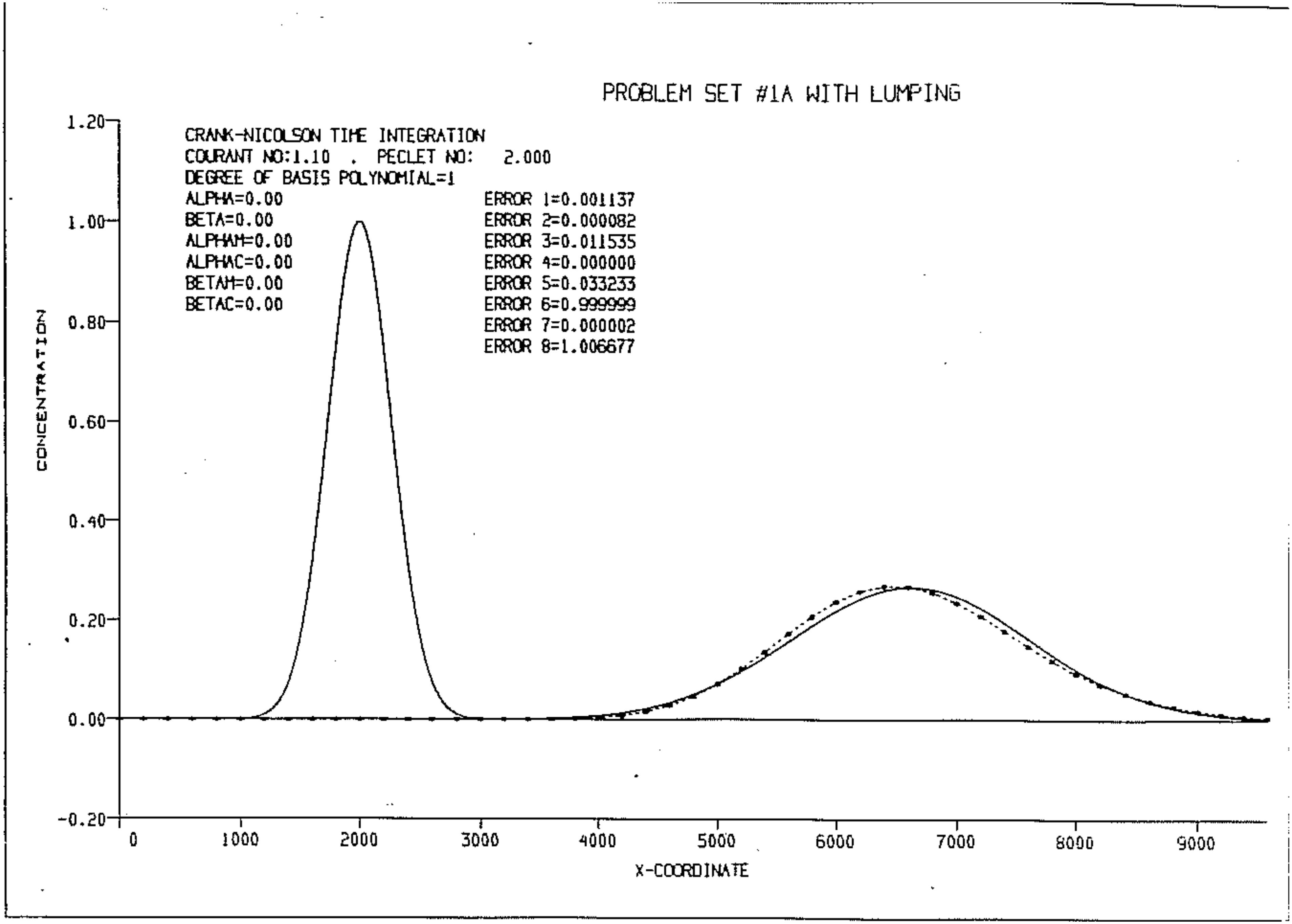

PROBLEM SET #1A WITH LUMPING
CRANK-NICOLSON TIME INTEGRATION
COURANT NO:1.10 . PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.001137
ERROR 2=0.000082
ERROR 3=0.011535
ERROR 4=0.000000
ERROR 5=0.033233
ERROR 6=0.999999
ERROR 7=0.000002
ERROR 8=1.006677
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.8b2

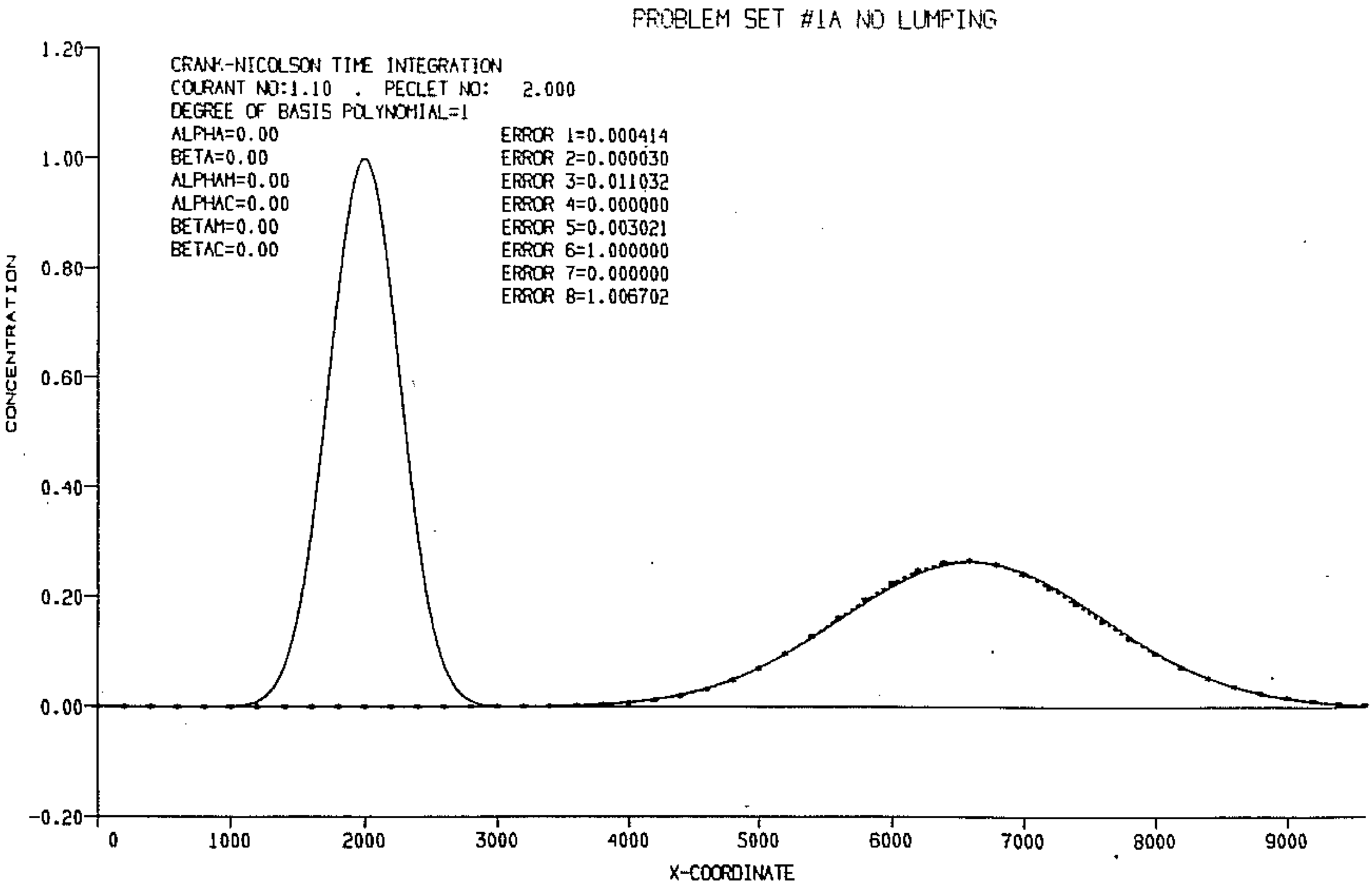

PROBLEM SET #1A NO LUMPING
CRANK-NICOLSON TIME INTEGRATION
COURANT NO:1.10 . PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.000414
ERROR 2=0.000030
ERROR 3=0.011032
ERROR 4=0.000000
ERROR 5=0.003021
ERROR 6=1.000000
ERROR 7=0.000000
ERROR 8=1.006702
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.9a1

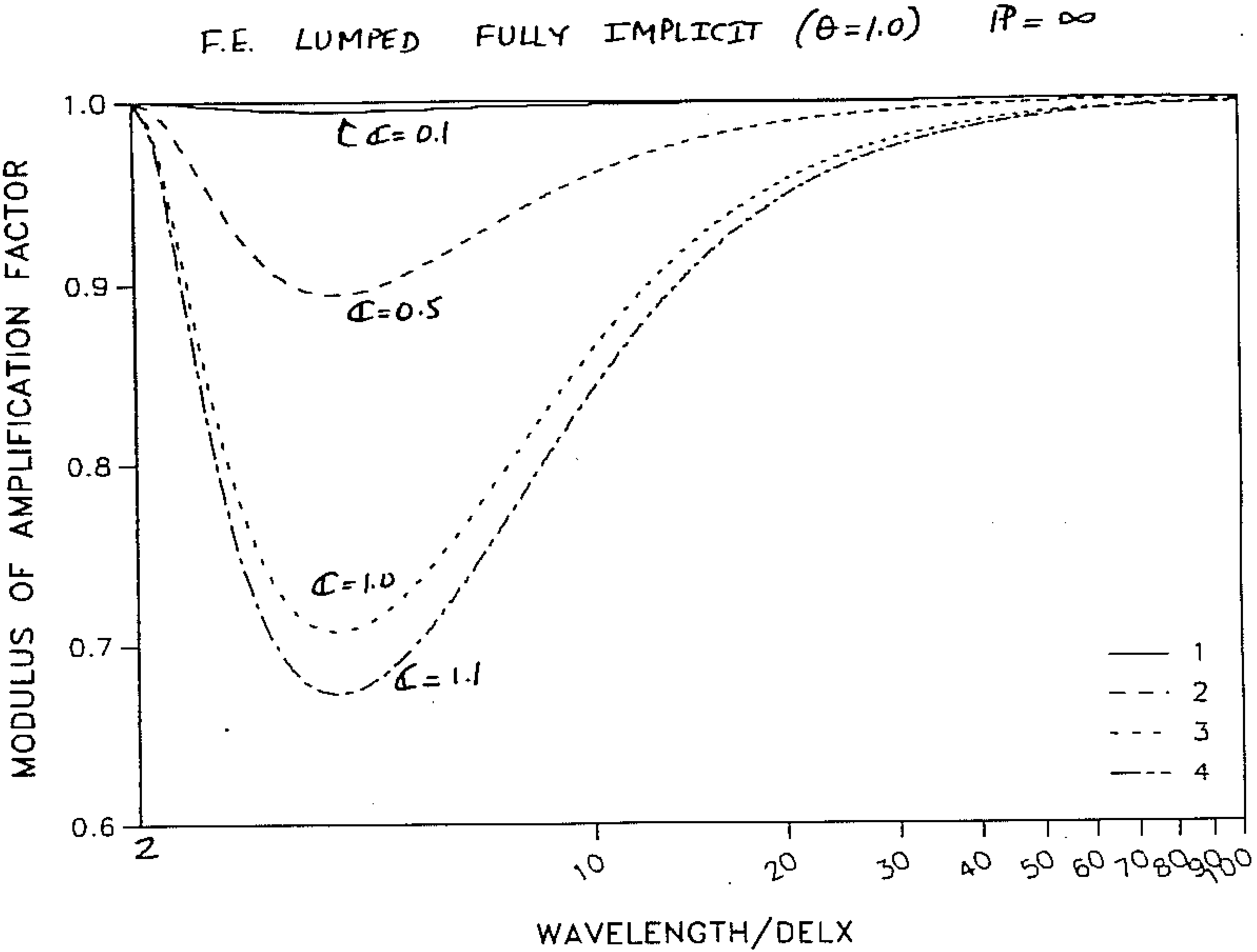

F.E. LUMPED FULLY IMPLICIT (θ=1.0) P = ∞
1.0
0.9
0.8
0.7
0.6
MODULUS OF AMPLIFICATION FACTOR
C = 0.1
C = 0.5
C = 1.0
C = 1.1
1
2
3
4
2
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.9a2

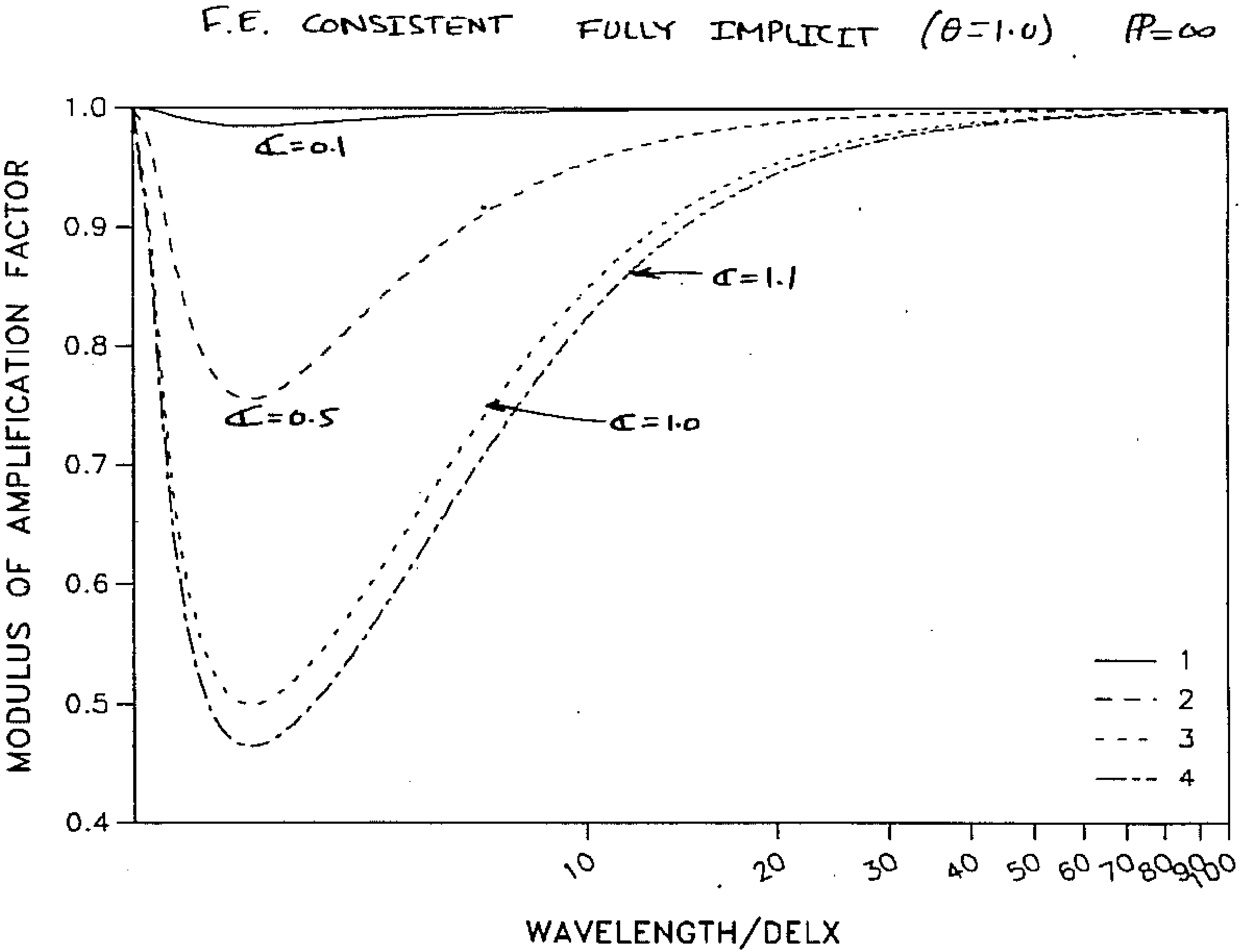

F.E. CONSISTENT FULLY IMPLICIT (θ=1.0) P=∞
MODULUS OF AMPLIFICATION FACTOR
WAVELENGTH/DELX
1.0
0.9
0.8
0.7
0.6
0.5
0.4
10
20
30
40
50
60
70
80
90
100
C=0.1
C=0.5
C=1.0
C=1.1
1
2
3
4

Fig. L29.9b1

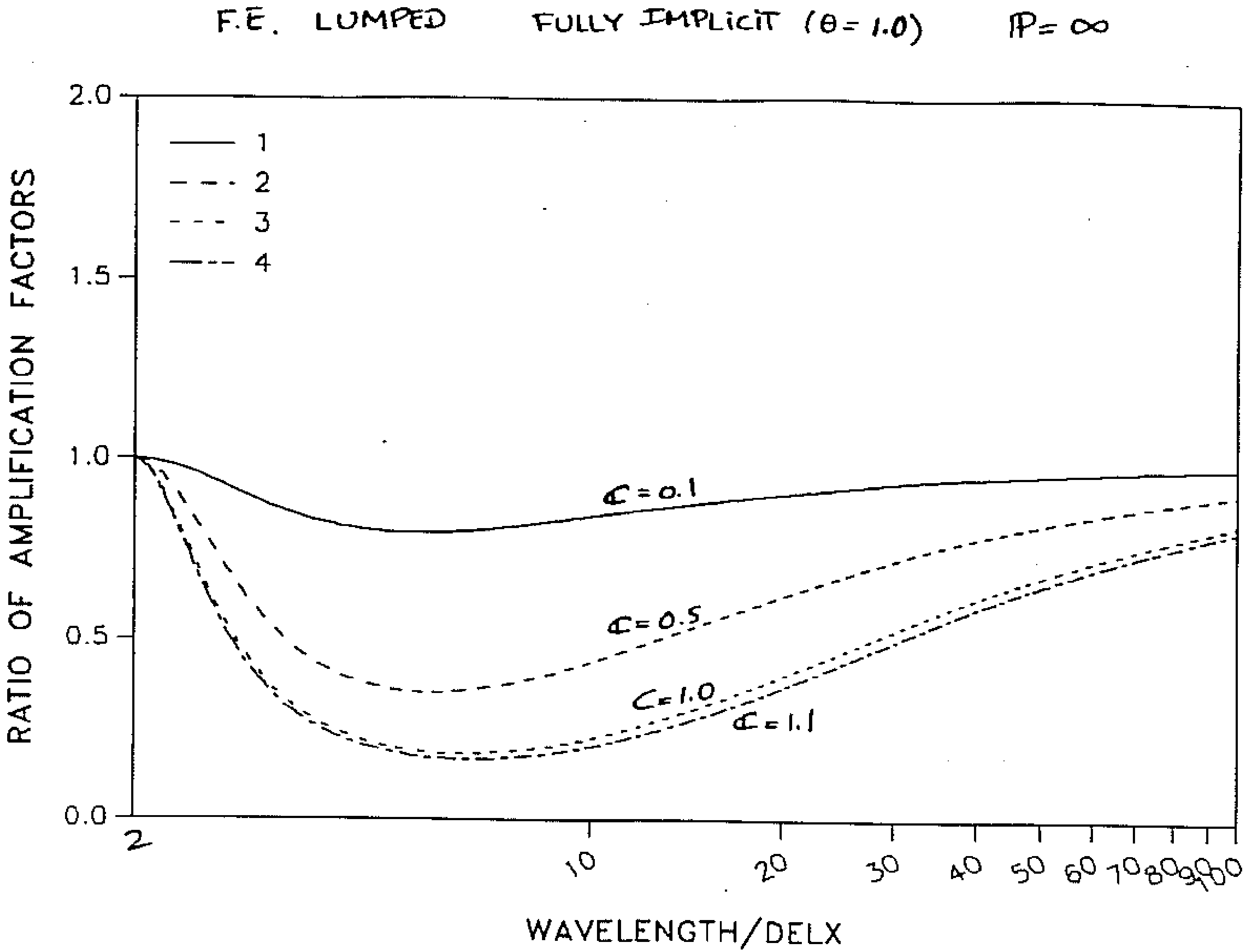

F.E. LUMPED FULLY IMPLICIT (θ = 1.0) P = ∞
2.0
1.5
1.0
0.5
0.0
1
2
3
4
RATIO OF AMPLIFICATION FACTORS
C = 0.1
C = 0.5
C = 1.0
C = 1.1
2
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.9b2

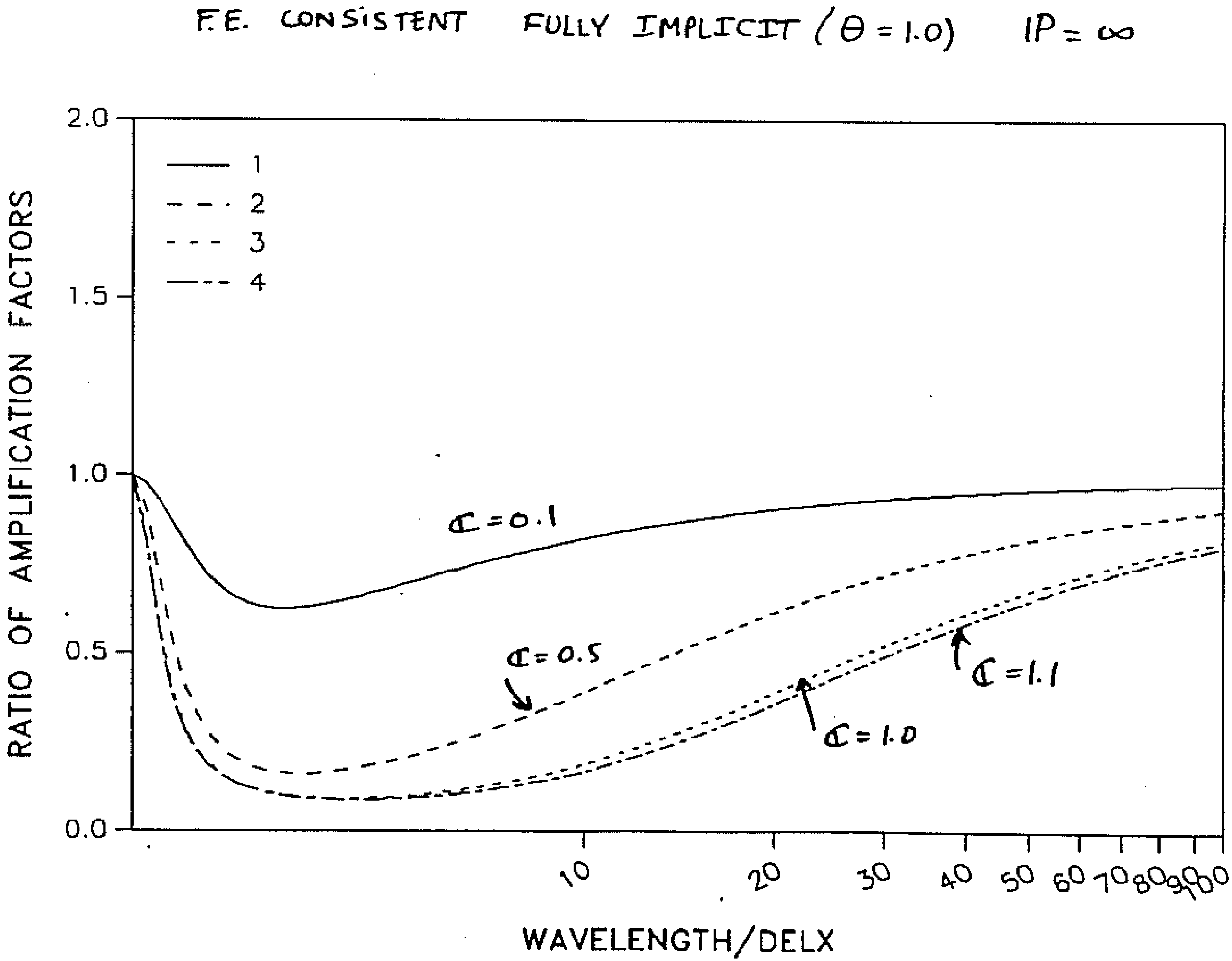
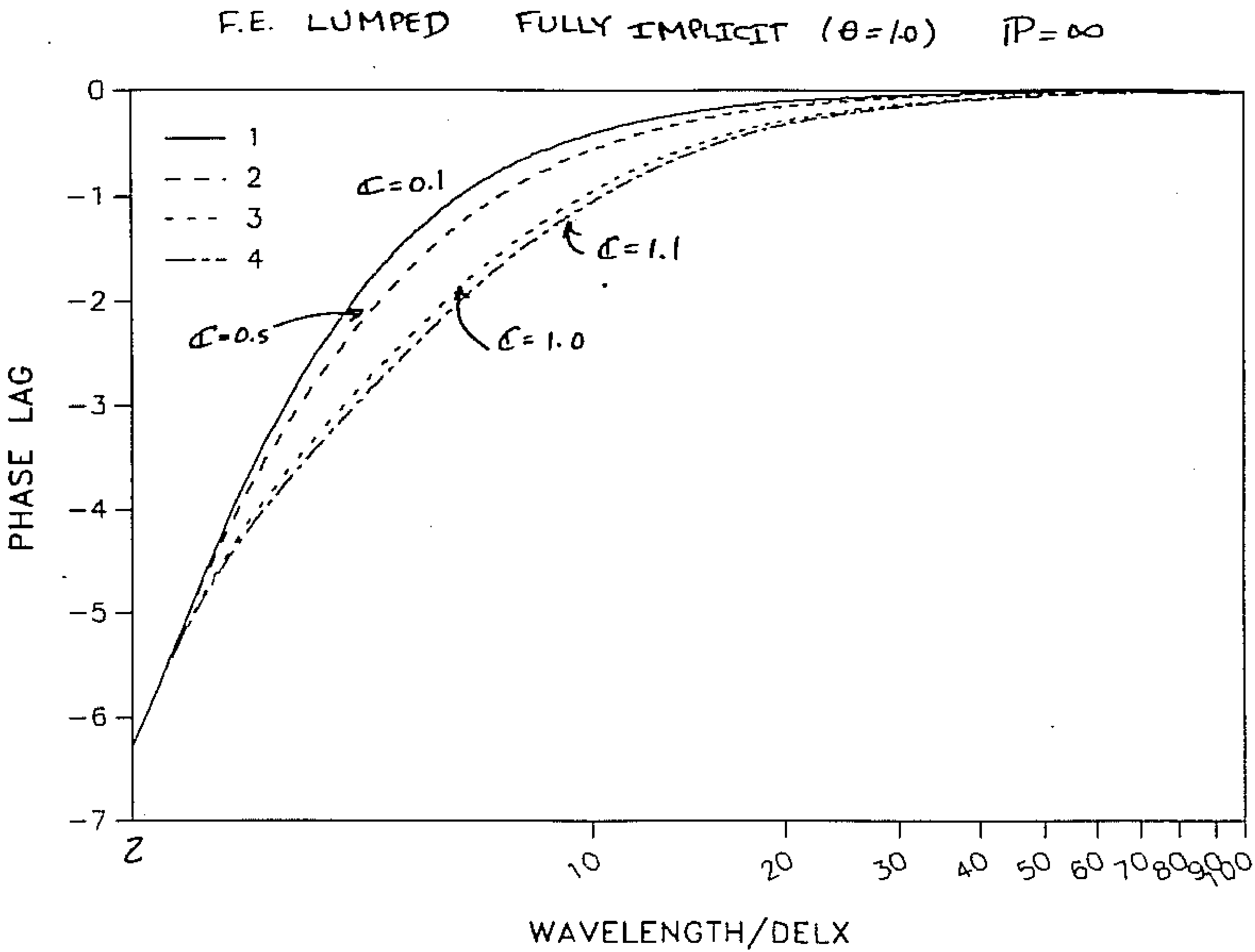
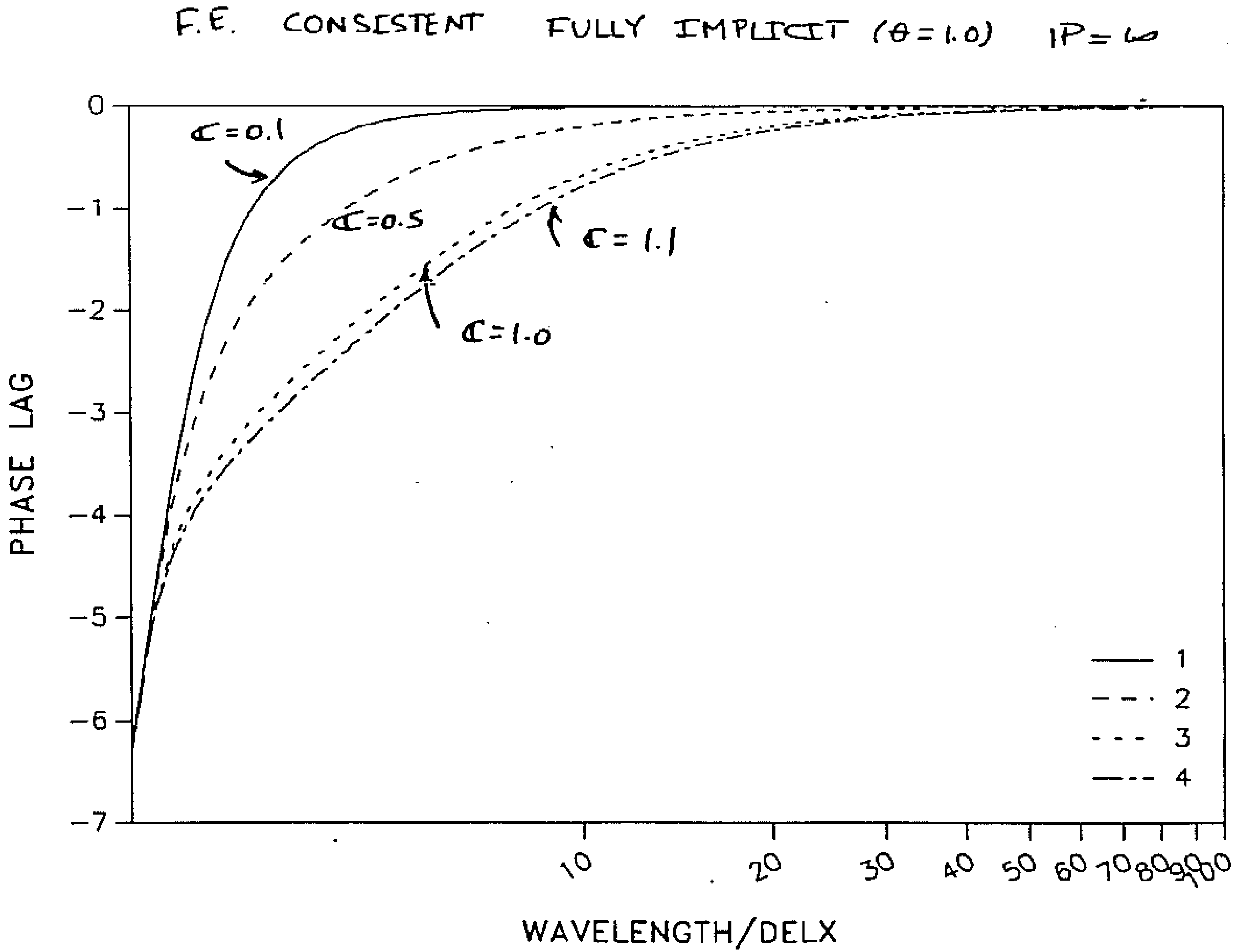

F.E. CONSISTENT FULLY IMPLICIT (θ = 1.0) IP = ∞
RATIO OF AMPLIFICATION FACTORS
2.0
1.5
1.0
0.5
0.0
1
2
3
4
C = 0.1
C = 0.5
C = 1.0
C = 1.1
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.9c1



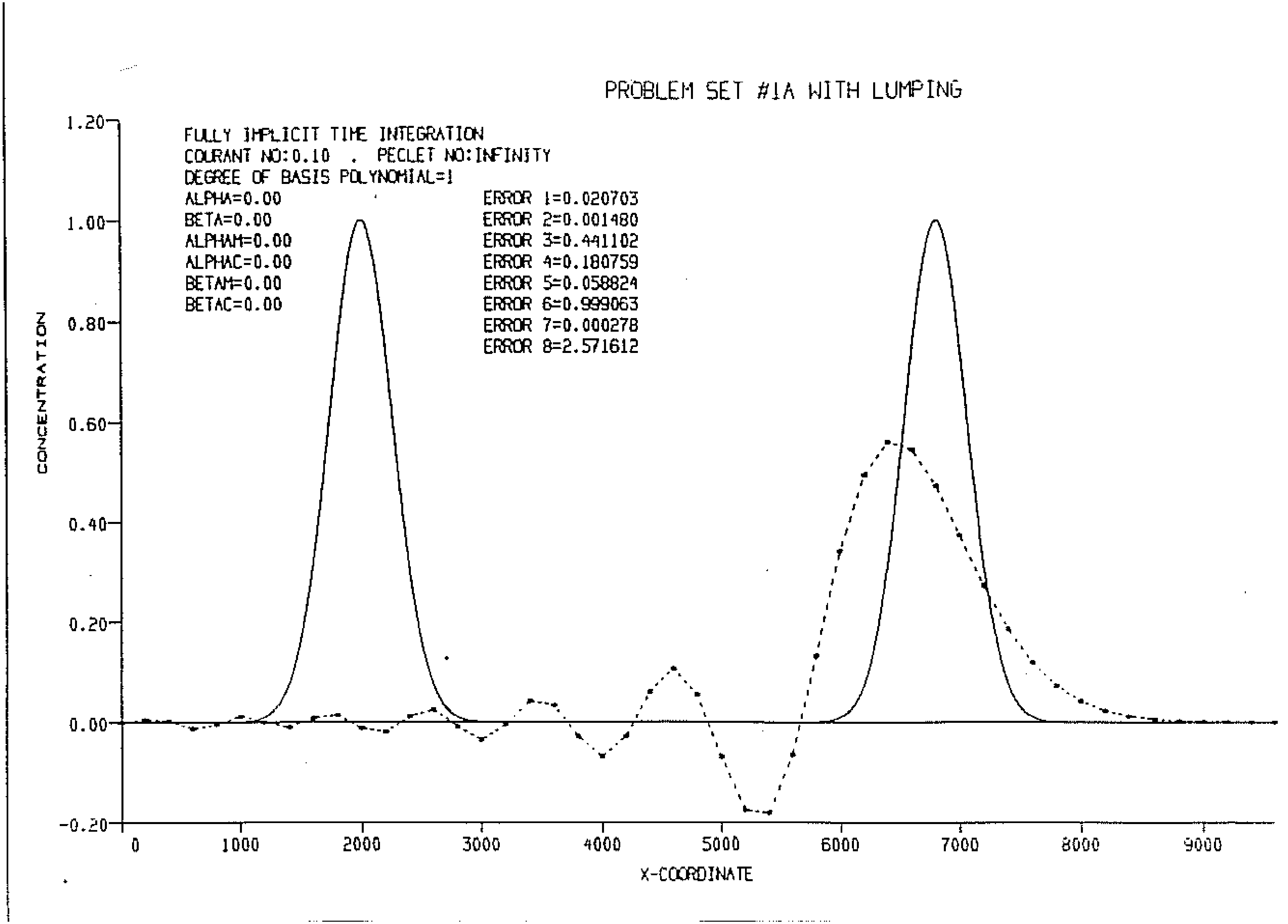
F.E. LUMPED FULLY IMPLICIT (θ=1.0) IP=∞
PHASE LAG
0
−1
−2
−3
−4
−5
−6
−7
1
2
3
4
C=0.1
C=0.5
C=1.1
C=1.0
2
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.9c2



F.E. CONSISTENT FULLY IMPLICIT (θ=1.0) IP=6
0
−1
−2
−3
−4
−5
−6
−7
PHASE LAG
C=0.1
C=0.5
C=1.1
C=1.0
1
2
3
4
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.10a1



PROBLEM SET #1A WITH LUMPING
FULLY IMPLICIT TIME INTEGRATION
COURANT NO:0.10 . PECLET NO:INFINITY
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.020703
ERROR 2=0.001480
ERROR 3=0.441102
ERROR 4=0.180759
ERROR 5=0.058824
ERROR 6=0.999063
ERROR 7=0.000278
ERROR 8=2.571612
CONCENTRATION
X-COORDINATE
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000

Fig. L29.10a2

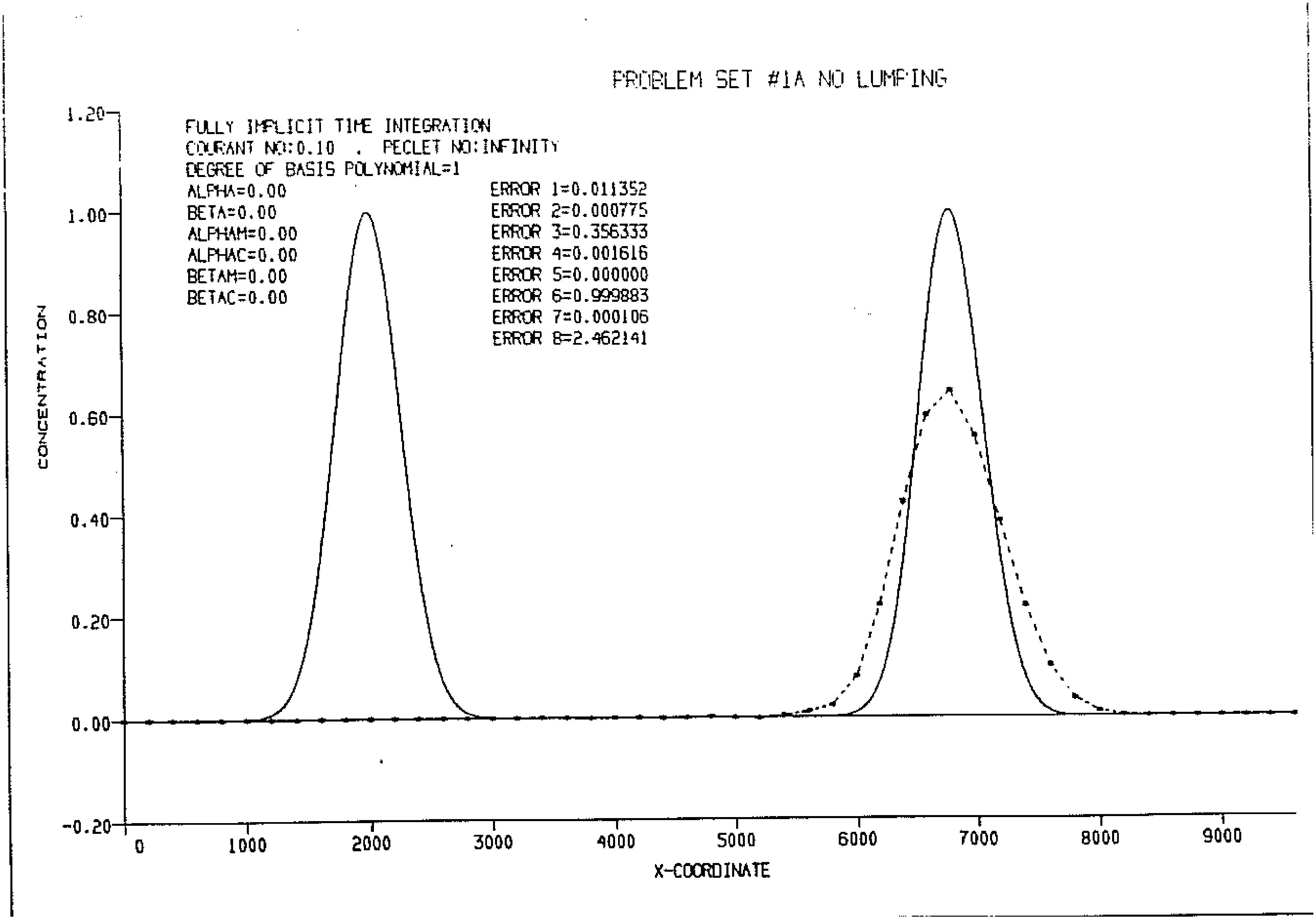
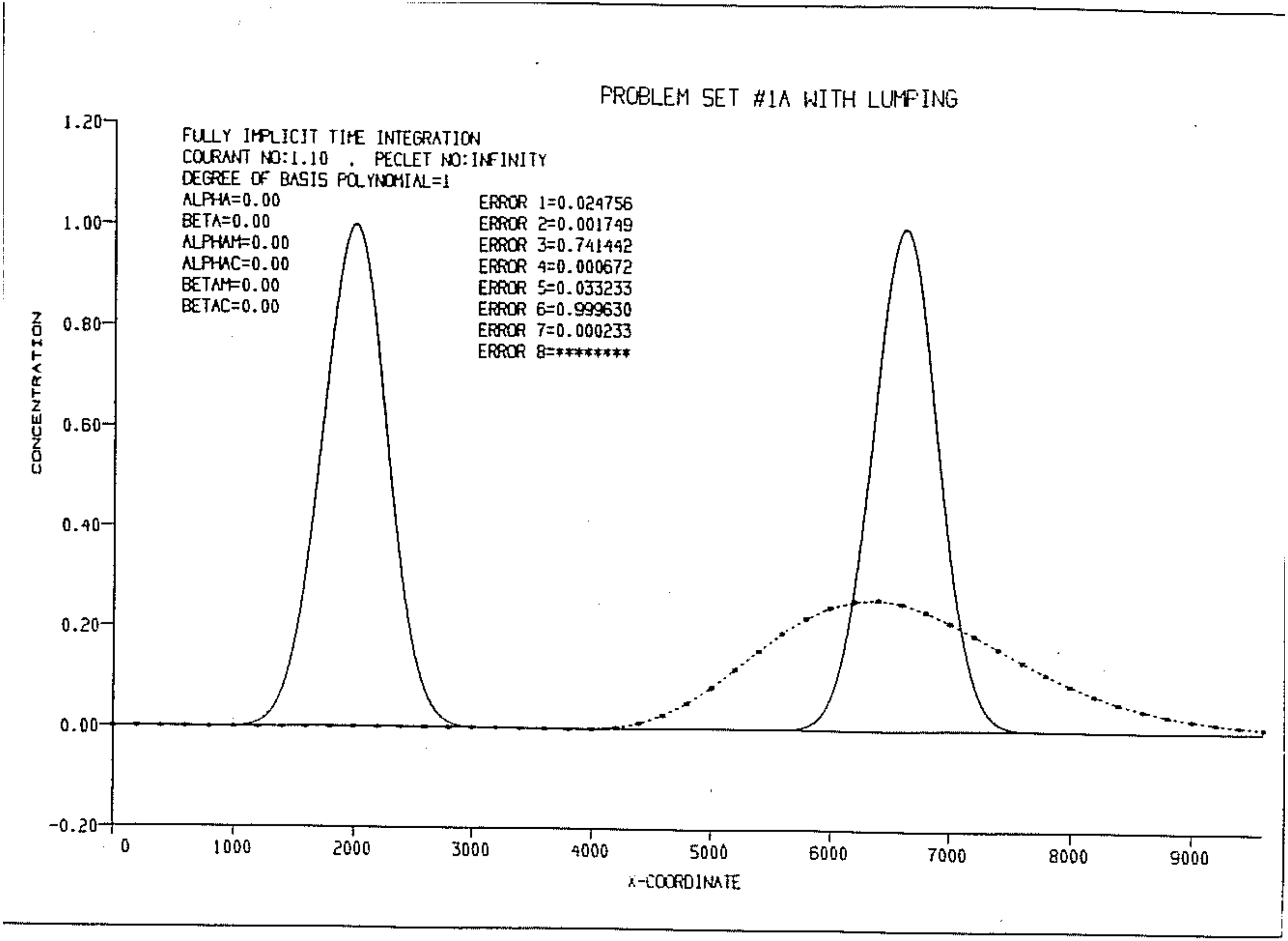

PROBLEM SET #1A NO LUMPING
FULLY IMPLICIT TIME INTEGRATION
COURANT NO:0.10 , PECLET NO:INFINITY
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.011352
ERROR 2=0.000775
ERROR 3=0.356333
ERROR 4=0.001616
ERROR 5=0.000000
ERROR 6=0.999883
ERROR 7=0.000106
ERROR 8=2.462141
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.10b1



PROBLEM SET #1A WITH LUMPING
FULLY IMPLICIT TIME INTEGRATION
COURANT NO:1.10 . PECLET NO:INFINITY
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.024756
ERROR 2=0.001749
ERROR 3=0.741442
ERROR 4=0.000672
ERROR 5=0.033233
ERROR 6=0.999630
ERROR 7=0.000233
ERROR 8=********
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.10b2

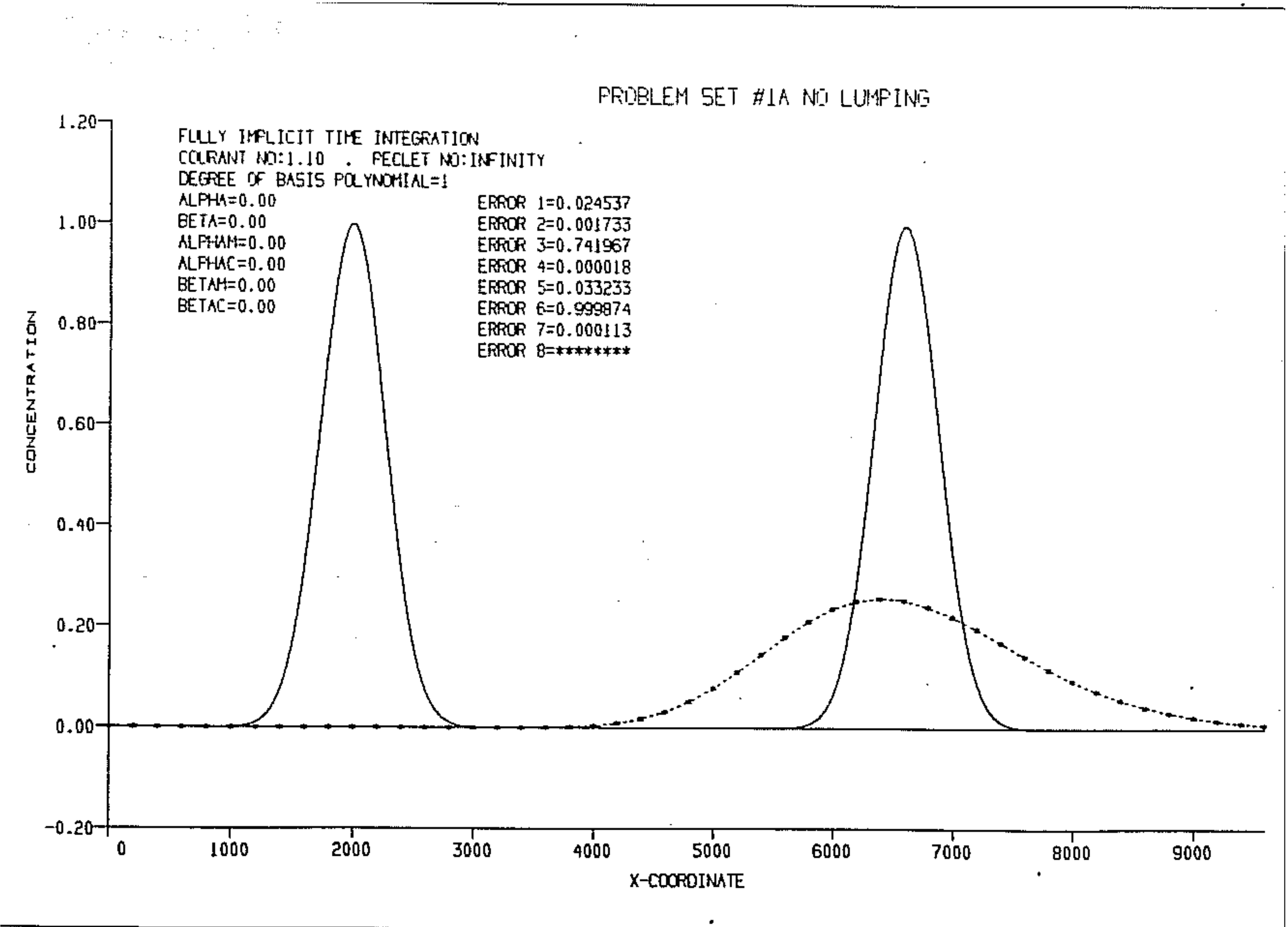
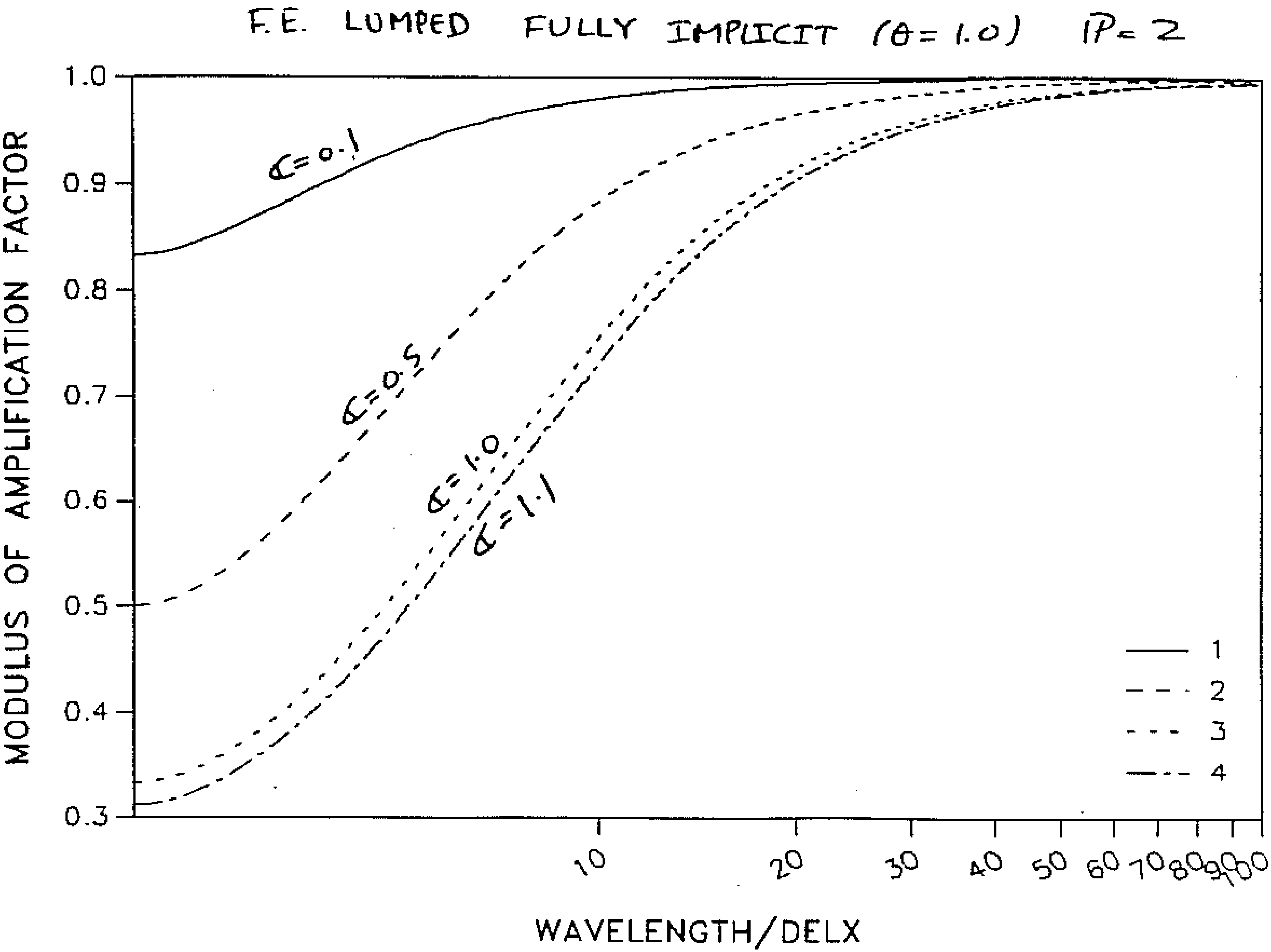

PROBLEM SET #1A NO LUMPING
FULLY IMPLICIT TIME INTEGRATION
COURANT NO:1.10 . PECLET NO:INFINITY
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.024537
ERROR 2=0.001733
ERROR 3=0.741967
ERROR 4=0.000018
ERROR 5=0.033233
ERROR 6=0.999874
ERROR 7=0.000113
ERROR 8=********
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.11a1



F.E. LUMPED FULLY IMPLICIT (θ = 1.0) IP = 2
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
MODULUS OF AMPLIFICATION FACTOR
α = 0.1
α = 0.5
α = 1.0
α = 1.1
1
2
3
4
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.11a2

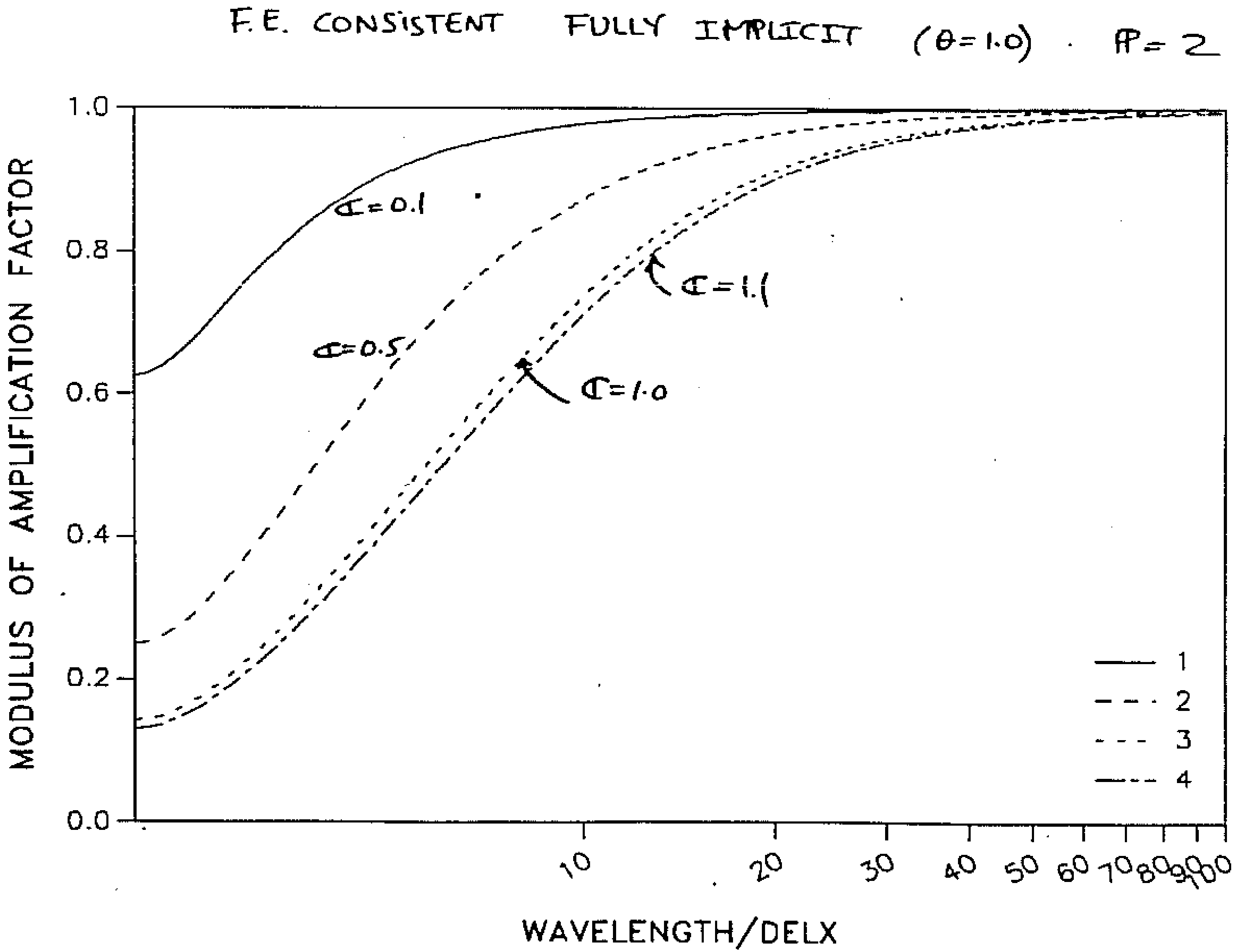

F.E. CONSISTENT FULLY IMPLICIT (θ=1.0) . P=2
MODULUS OF AMPLIFICATION FACTOR
1.0
0.8
0.6
0.4
0.2
0.0
C=0.1
C=0.5
C=1.1
C=1.0
1
2
3
4
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.11b1

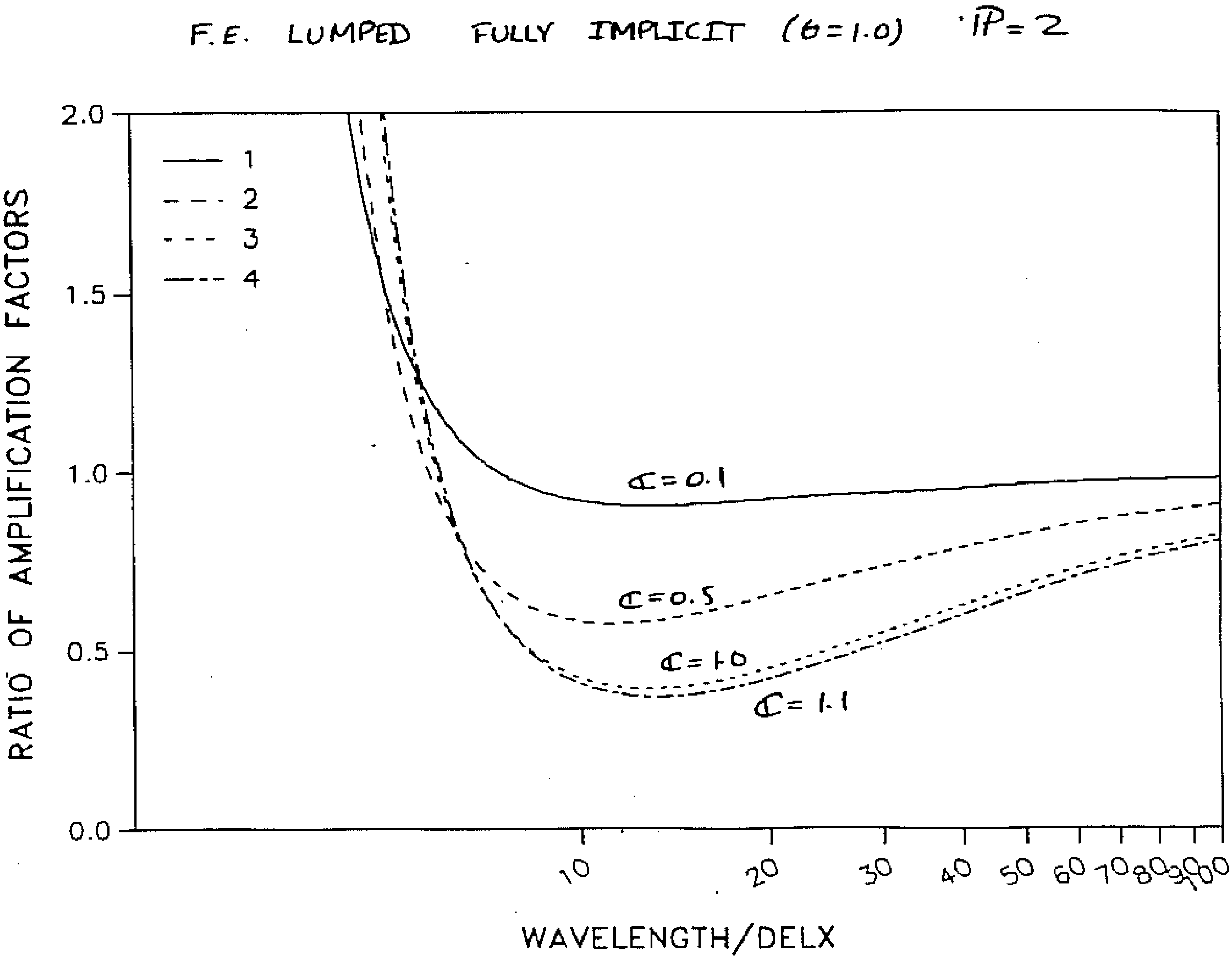

F.E. LUMPED FULLY IMPLICIT (θ=1.0) IP=2
2.0
1.5
1.0
0.5
0.0
1
2
3
4
RATIO OF AMPLIFICATION FACTORS
C=0.1
C=0.5
C=1.0
C=1.1
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.11b2

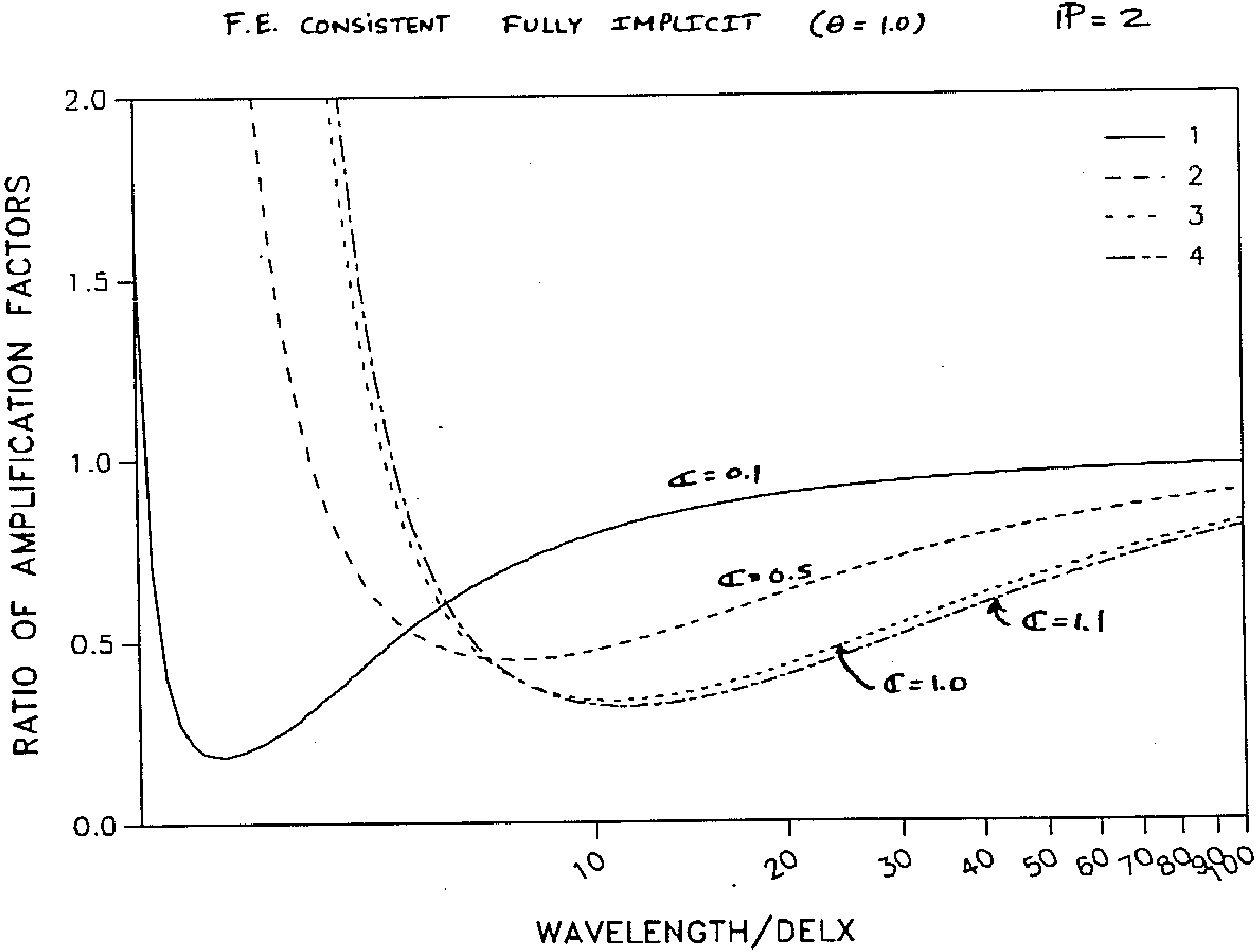
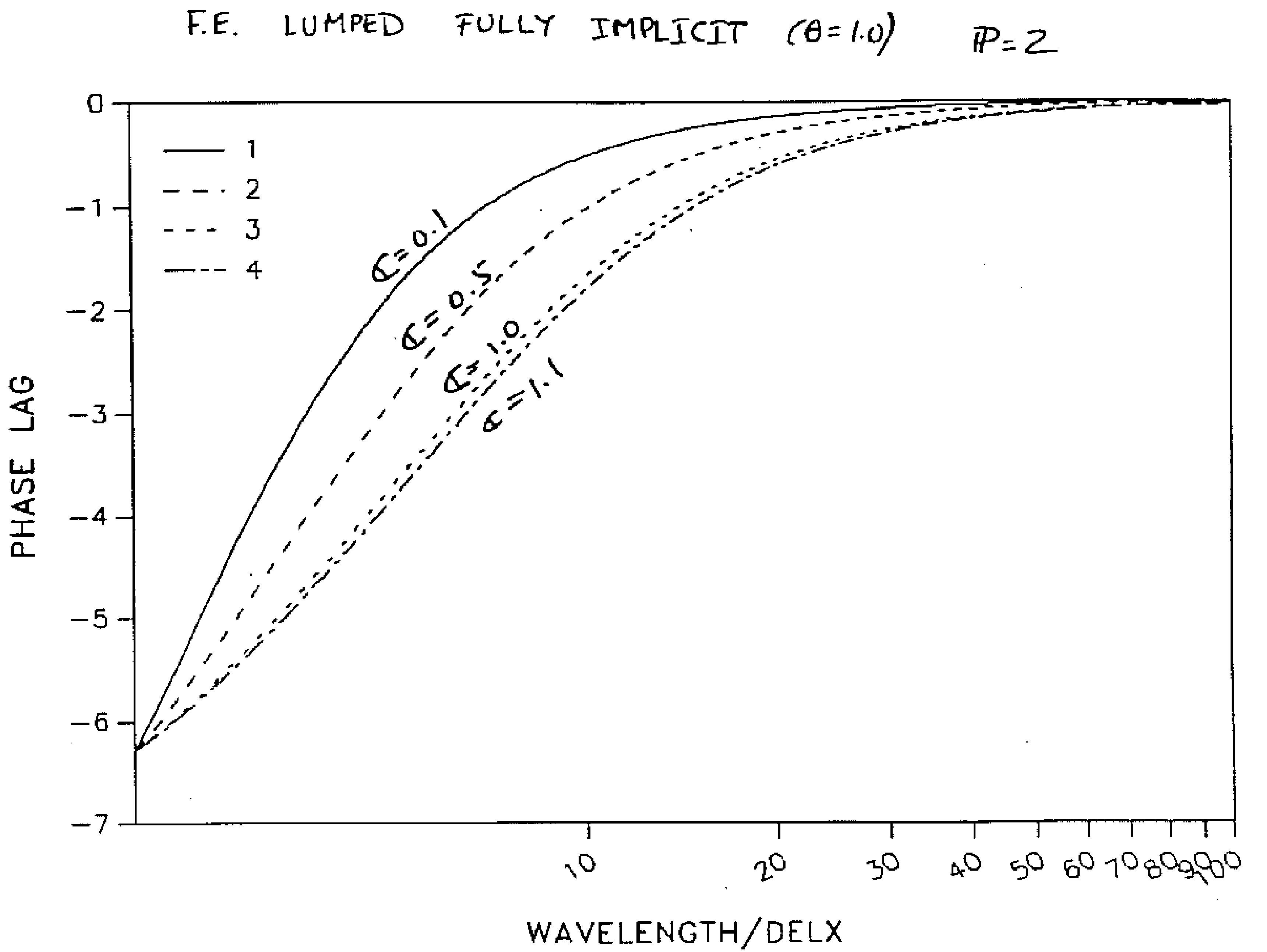

F.E. CONSISTENT FULLY IMPLICIT (θ = 1.0) P = 2
RATIO OF AMPLIFICATION FACTORS
2.0
1.5
1.0
0.5
0.0
WAVELENGTH/DELX
10
20
30
40
50
60
70
80
90
100
1
2
3
4
C = 0.1
C = 0.5
C = 1.1
C = 1.0

Fig. L29.11c1



F.E. LUMPED FULLY IMPLICIT (θ=1.0) P=2
0
−1
−2
−3
−4
−5
−6
−7
1
2
3
4
C=0.1
C=0.5
C=1.0
C=1.1
PHASE LAG
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.11c2

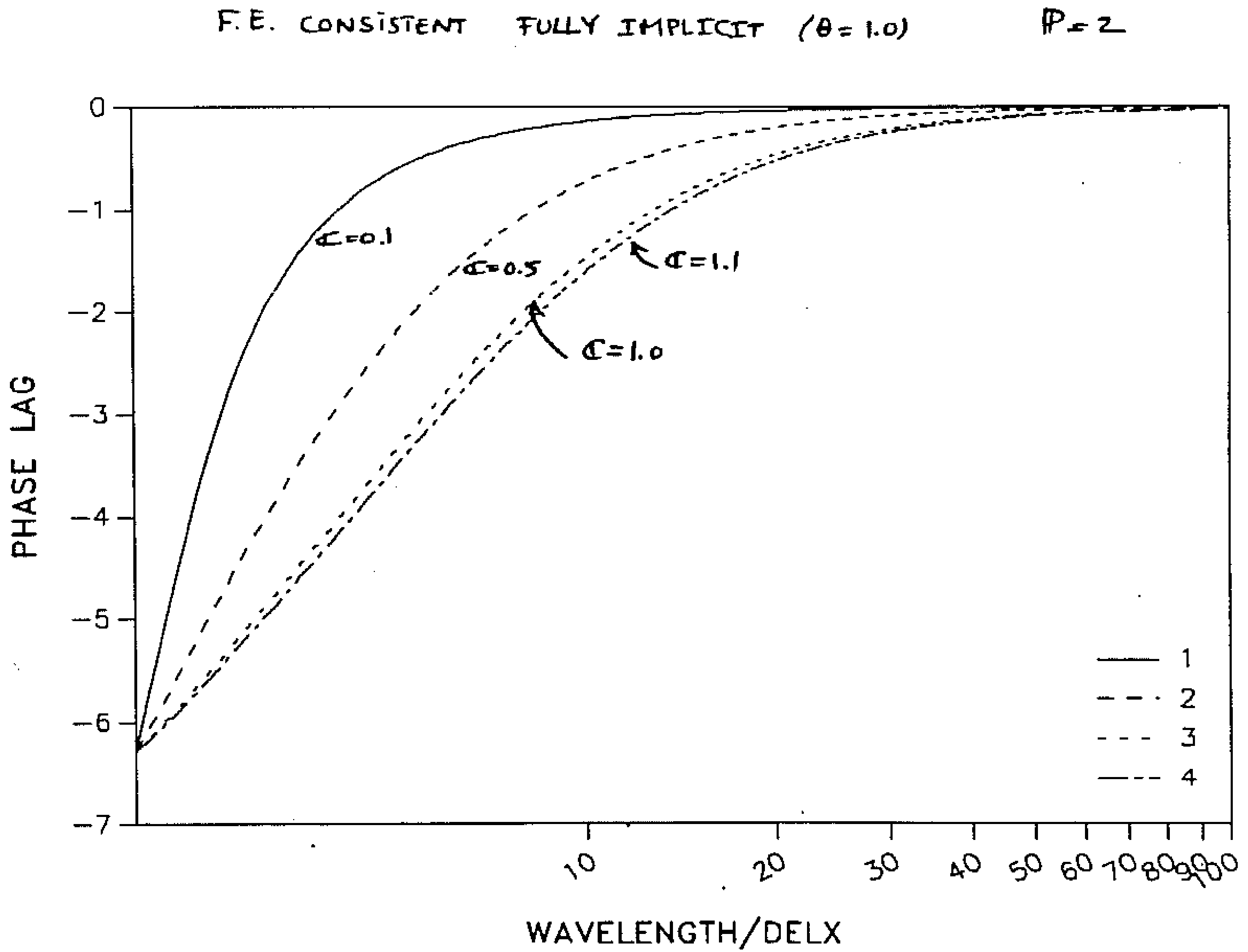

F.E. CONSISTENT FULLY IMPLICIT (θ = 1.0) ℙ = 2
PHASE LAG
0
−1
−2
−3
−4
−5
−6
−7
C=0.1
C=0.5
C=1.1
C=1.0
1
2
3
4
10
20
30
40
50
60
70
80
90
100
WAVELENGTH/DELX

Fig. L29.12a1

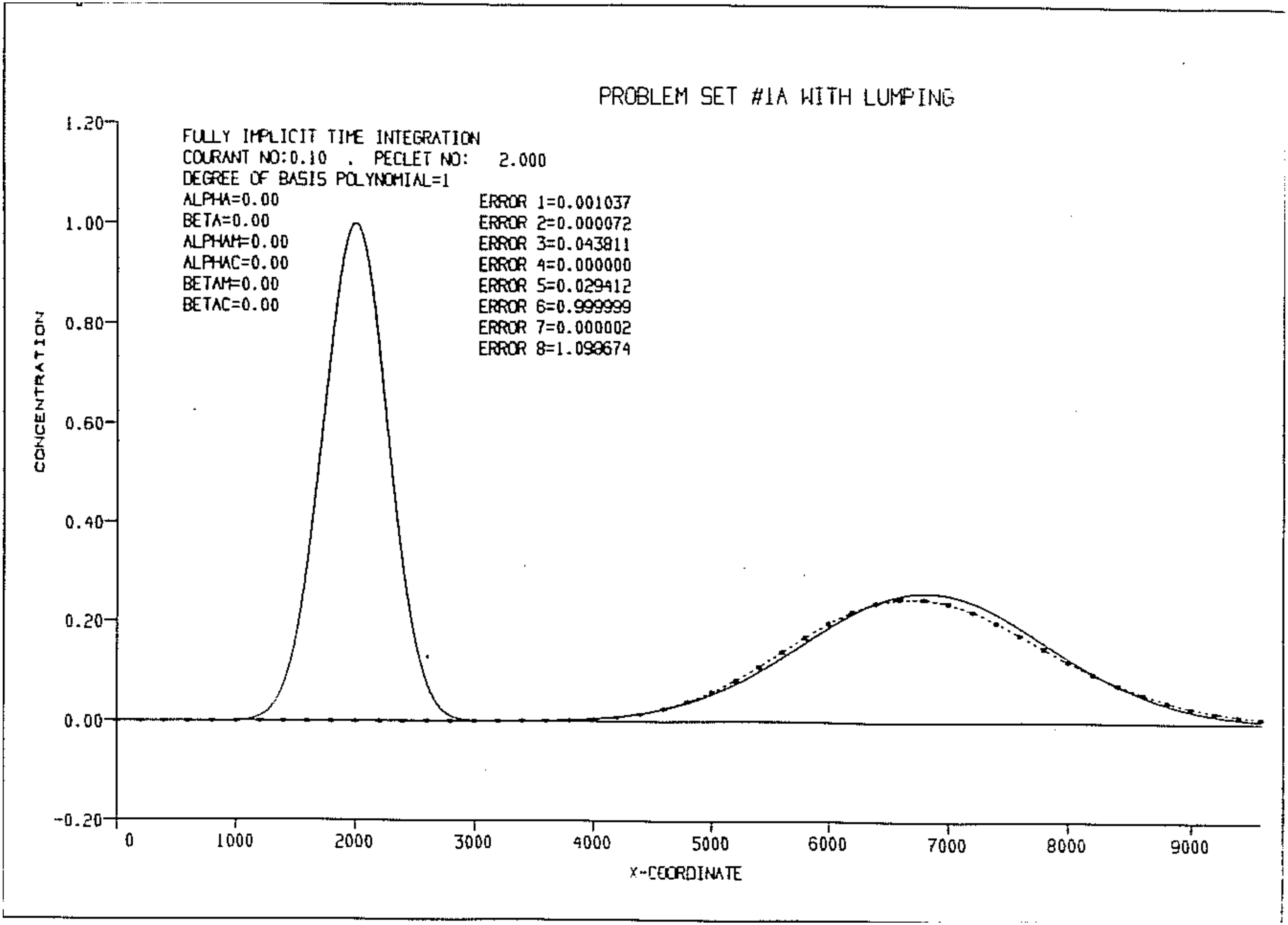

PROBLEM SET #1A WITH LUMPING
FULLY IMPLICIT TIME INTEGRATION
COURANT NO:0.10 . PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.001037
ERROR 2=0.000072
ERROR 3=0.043811
ERROR 4=0.000000
ERROR 5=0.029412
ERROR 6=0.999999
ERROR 7=0.000002
ERROR 8=1.098674
CONCENTRATION
X-COORDINATE
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000

Fig. L29.12a2

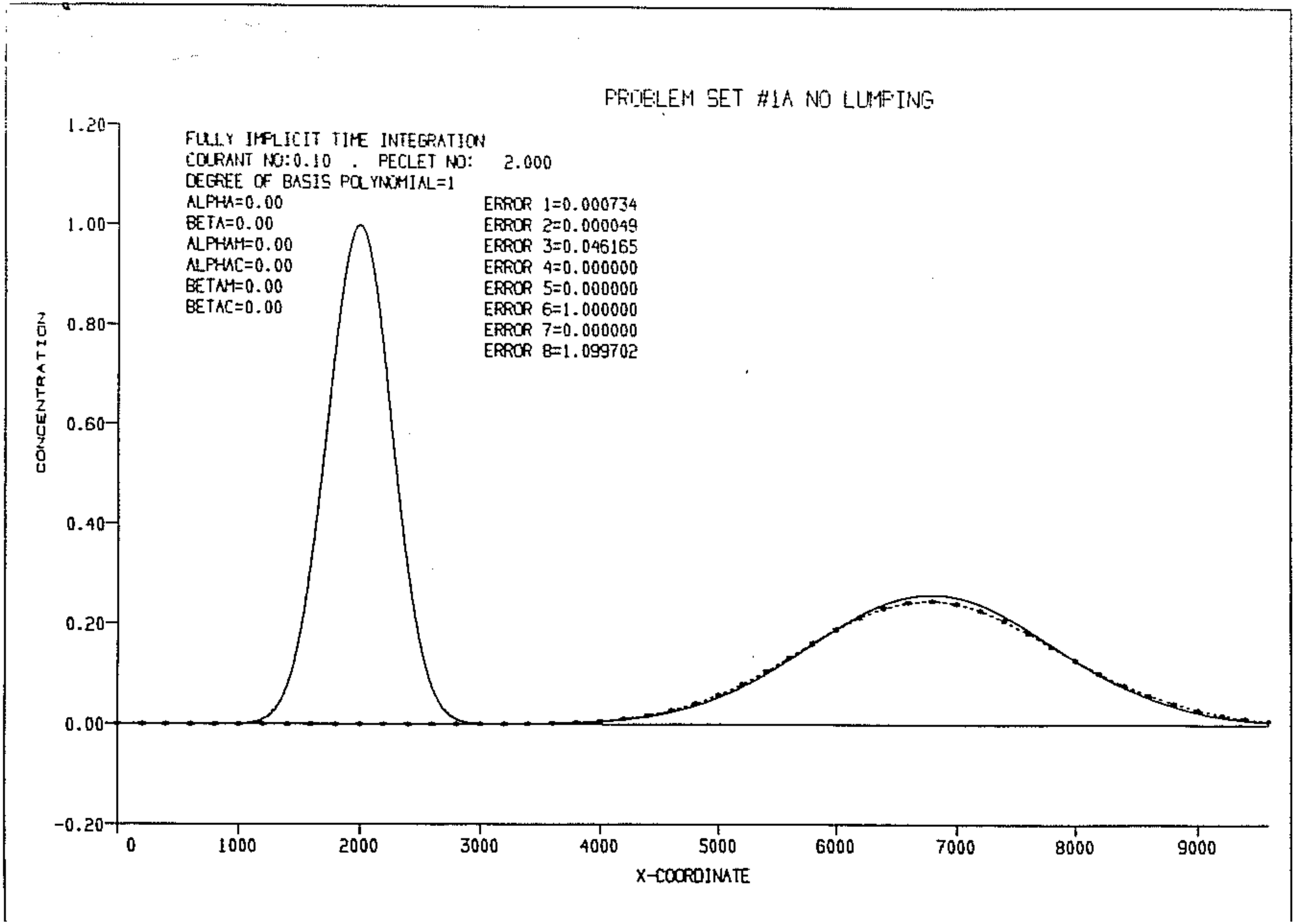

PROBLEM SET #1A NO LUMPING
FULLY IMPLICIT TIME INTEGRATION
COURANT NO:0.10 . PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.000734
ERROR 2=0.000049
ERROR 3=0.046165
ERROR 4=0.000000
ERROR 5=0.000000
ERROR 6=1.000000
ERROR 7=0.000000
ERROR 8=1.099702
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.12b1

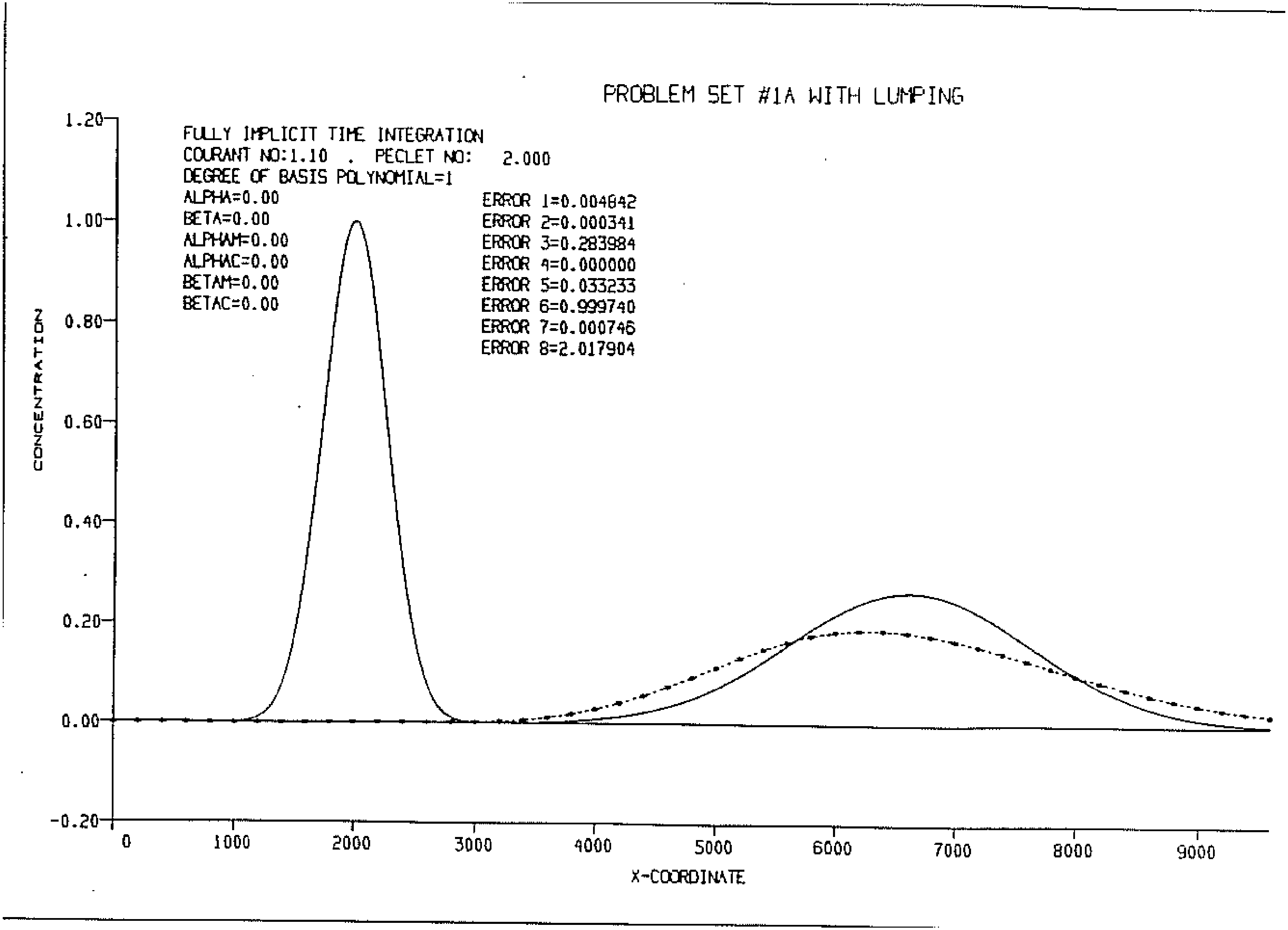
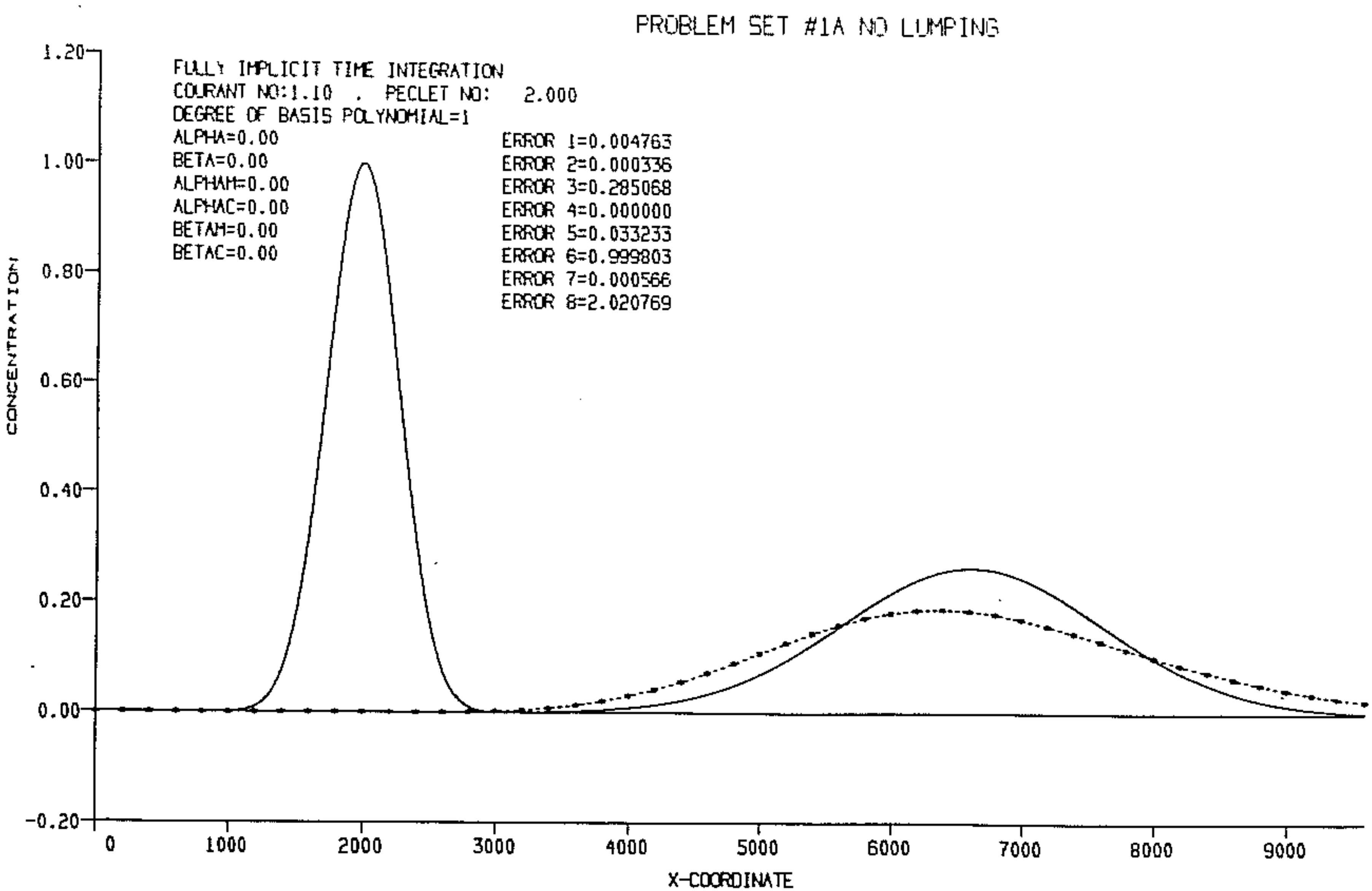

PROBLEM SET #1A WITH LUMPING
FULLY IMPLICIT TIME INTEGRATION
COURANT NO:1.10 . PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.004842
ERROR 2=0.000341
ERROR 3=0.283984
ERROR 4=0.000000
ERROR 5=0.033233
ERROR 6=0.999740
ERROR 7=0.000746
ERROR 8=2.017904
CONCENTRATION
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
X-COORDINATE

Fig. L29.12b2


PROBLEM SET #1A NO LUMPING
FULLY IMPLICIT TIME INTEGRATION
COURANT NO:1.10 . PECLET NO: 2.000
DEGREE OF BASIS POLYNOMIAL=1
ALPHA=0.00
BETA=0.00
ALPHAM=0.00
ALPHAC=0.00
BETAM=0.00
BETAC=0.00
ERROR 1=0.004763
ERROR 2=0.000336
ERROR 3=0.285068
ERROR 4=0.000000
ERROR 5=0.033233
ERROR 6=0.999803
ERROR 7=0.000566
ERROR 8=2.020769
CONCENTRATION
X-COORDINATE
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0
1000
2000
3000
4000
5000
6000
7000
8000
9000